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A tough nut to crack- Series problem 1:

We have been asked to find the sum of the following infinite series of triangular numbers:

$$s = 1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \cdots$$

So if we were to divide s by 2 we would gain the series

$$\frac{s}{2} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \cdots$$

Each term can then be written as a small finite sum in its own right:

$$\frac{s}{2} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \cdots$$

This allows us to sneakily cancel all the terms after the 1, shown more clearly by removing the brackets below.

$$\frac{s}{2} = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} + \cdots = 1$$

Now that that we know $\frac{s}{2} = 1$, it gives that s must equal 2. So the sum of this series of triangular number is $s = 2$.