Exercises **I.1**

Brief solutions end of Exercises.

Complete solutions at www.oup.co.uk/companion/NumberTheory

1. Which of the following are propositions?

The ones that are propositions state whether they are true or false.

(a) $2+2=4$

(b) $2+2=3$

(c) All Swedish subjects have blonde hair.

(d) She looks beautiful.

(e) $x^2-1=0$

2. Negate the following propositions:

(i) Man can be pregnant.

(ii) Grass is green.

(iii) Lecturers annual salary is over £45 000.

(iv) There are integers a and b such that $\frac{a}{b}=\pi$.

(v) There are integers a and b such that $\frac{a}{b}=e$.

3. Write the following in words:

$$x^2-9=0 \Rightarrow x^2=9$$
$$\Rightarrow x=\sqrt{9}$$
$$\Rightarrow x=\pm 3$$

4. Let $P: x<3$, $\quad Q: x^2<9$. Write a sentence for

(i) $P \Rightarrow Q$ (ii) $Q \Rightarrow P$

Do you think either of them, (i) and (ii), is true?

5. Let P: ABC is an equilateral triangle.

Q: All the angles inside the triangle ABC are equal.

Write a sentence for

(i) $P \Rightarrow Q$ (ii) $Q \Rightarrow P$

Are both of these, (i) and (ii), true?

6. Let P : n is prime and Q : $2^n - 1$ is prime. Write the following in words:

(i) $P \Rightarrow Q$ (ii) $Q \Rightarrow P$

*Do you think either of them, (i) and (ii), is true?

7. Construct the truth table for $Q \vee P$. Compare your answer with the truth value for $P \vee Q$. What do you notice?

8. Construct the truth table for $Q \wedge P$. What do you notice about your result by comparing with $P \wedge Q$?

9. Construct the truth table for:

(a) $\left(\neg P\right) \wedge P$ (b) $\left(\neg P\right) \vee P$ (c) $\neg\left(\neg P\right)$

10. What are the following compound propositions equivalent to?

(a) $\neg\left(\neg\left(\neg P\right)\right)$ (b) $P \wedge P$ (c) $P \vee P$ (d) $\left(\neg P\right) \wedge \left(\neg P\right)$

11. Show that $\neg\left(P \wedge Q\right)$ and $\neg P \vee \neg Q$ are equivalent.

12. Show that $P \wedge \left(Q \vee R\right)$ and $\left(P \wedge Q\right) \vee \left(P \wedge R\right)$ are equivalent.

13. Devise an equivalence rule to the following compound proposition:

$$\neg\neg \ \ldots \ \left(\neg P\right)$$

[Hint: The rule will depend on the number of $\neg$]

Let P be the proposition 'the cup is full'. By using this devised rule what is $\neg\neg\neg\neg P$ equal to?

14. Construct the truth table for $\left(\neg P\right) \Rightarrow Q$.

15. Construct the truth table for

$$\left[\left(P \wedge \left(\neg Q\right)\right) \Rightarrow \left(R \wedge \left(\neg R\right)\right)\right] \Rightarrow \left(P \Rightarrow Q\right)$$

What do you notice about your results?

Brief Solutions to Exercises I.1

1. (a), (b) and (c). 4 and 5. Both Proposition are true. 6. Only (ii) is true.