Proper factors of 6 are 1, 2 and 3.

$1+2+3=6$. ✓

" " " 28 are 1, 2, 4, 7, 14

$1+2+4+7+14=\underline{\underline{28}}$

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Th (4.28) Page 197

$N = 2^{p-1}(\underbrace{2^p-1}_{=\text{prime}})$ is a perfect number.

Proof: We use the geometric series

(4.29) $a + ar + ar^2 + \cdots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$

$1 = 2^0, 2^1, 2^2, 2^3, \ldots, 2^{p-1}$

$(2^p-1), 2(2^p-1), 2^2(2^p-1), \ldots, 2^{p-2}(2^p-1)$ *

Don't include $N = 2^{p-1}(2^p-1)$

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-1} = \frac{1(1-2^p)}{1-2}$$

$$= -(1-2^p)$$

$$= 2^p - 1 \quad (+)$$

$(2^p-1).1 + 2(2^p-1) + 2^2(2^p-1) + \cdots + 2^{p-2}(2^p-1)$

$$= (2^p-1)\left[\underbrace{1 + 2 + 2^2 + 2^3 + \cdots + 2^{p-2}}_{\text{// Geometric Series.}}\right]$$

$$= (2^p-1)\frac{1.(1-2^{p-1})}{1-2}$$

$$= (2^p - 1)\left[-(1 - 2^{p-1})\right]$$

$$= (2^p - 1)\left[2^{p-1} - 1\right]$$

Collecting all the proper factors gives

$$(2^p - 1).1 + (2^p - 1)\left[2^{p-1} - 1\right]$$

$$= (2^p - 1)\left[1 + 2^{p-1} - 1\right]$$

$$= 2^{p-1}\left[2^p - 1\right] = N.$$

$\Rightarrow$ N is a perfect number.

## Sigma Function Page 200.

Example 4.22

$$\sigma(10) = 1+2+5+10 = 18$$

$$\sigma(12) = 1+2+3+4+6+12 = 28$$

$$\sigma(28) = \underbrace{1+2+4+7+14}_{28} + 28 = 56$$

$$\sigma(31) = 1+31 = 32.$$

Predict

$$\sigma(p) = p+1$$

Proposition (4.33)

Let n be a perfect number then

$$\sigma(n) = 2n.$$

Proof: Let $d_1, d_2, \ldots, d_k$ be the proper factors of $n$.

$$\sigma(n) = \underbrace{d_1 + d_2 + d_3 + \cdots + d_k}_{=n} + n = n+n = 2n.$$

$\square$

---

$$\sigma(561)$$

Soln:

$$561 = 3 \times 11 \times 17.$$

$$\sigma(561) = \sigma(3 \times 11 \times 17) \qquad \sigma(p) = p+1$$

$$= \sigma(3) \times \sigma(11) \times \sigma(17)$$

$$= 4 \times 12 \times 18 = 864$$

$\sigma(n) < 2n$ then $n$ is deficient number.

Proposition (4.35)

$$\sigma(p^k) = \frac{p^{k+1}-1}{p-1}$$

Proof The factors of $p^k$ are
$1, p, p^2, p^3, \ldots, p^{k-1}$ and $p^k$.

$$\sigma(p^k) = 1 + p + p^2 + p^3 + \cdots + p^{k-1} + p^k$$

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$= \frac{1(1-p^{k+1})}{1-p}$$

$$= \frac{p^{k+1}-1}{p-1}.$$

$\square$

Example 4.24 Page 203

$\sigma(945)$.

Soln:

$$945 = 3^3 \times 5 \times 7$$

$$\sigma(945) = \sigma(3^3 \times 5 \times 7)$$

$$= \sigma(3^3) \times \sigma(5) \times \sigma(7)$$

$$= \left(\frac{3^{3+1}-1}{3-1}\right) \times 6 \times 8 = 1920.$$