Ex 3.4 Page 113

$$15x \equiv 45 \pmod{10}$$

Soln,

$$\cancel{15}x \equiv \cancel{15} \times 3 \pmod{10}$$

The gcd $(15, 10) = 5$.

(3.10) $\boxed{ac \equiv bc \pmod{n} \Rightarrow a \equiv b \left(\bmod \frac{n}{g}\right)}$

$$x \equiv 3 \bmod \left(\frac{10}{5}\right) \equiv 3 \pmod{2} \equiv 1 \pmod{2}$$

x is one more than a multiple of 2

$$x = 1 + 2k$$

$$12x \equiv 36 \pmod{11}$$

$$\cancel{12}x \equiv 3 \times \cancel{12} \pmod{11}$$

The gcd $(12, 11) = 1$ therefore

$$x \equiv 3 \pmod{11}$$

$$x = 3 + 11t$$

Prop (3.10) If $ac \equiv bc \pmod{n} \Rightarrow a \equiv b \pmod{\frac{n}{g}}$
where $g = \gcd(c, n)$.

Proof Let $g = \gcd(c, n)$.

$gx = c$ and $gy = n$ (*)

R.t.p. $\boxed{\frac{n}{g} \mid (a-b)}$

we are given $ac \equiv bc \pmod{n} \Rightarrow$

$$ac - bc = kn$$
$$(a-b)c = kn$$
$$(a-b)\cancel{g}x = k\cancel{g}y$$
$$(a-b)x = ky$$

$\Rightarrow y \mid (a-b)x$. (+)

We want $y \mid (a-b)$.

From (*) we have $x = \frac{c}{g}$ and $y = \frac{n}{g}$ and
$\gcd\left(\frac{c}{g}, \frac{n}{g}\right) = 1$. Why?

If $\gcd(a, b) = g$ then $\gcd\left(\frac{a}{g}, \frac{b}{g}\right) = 1$. 1.5

In (+) we have $y \mid (a-b)x$ & $\gcd(y, x) = 1$

By Euclid's Lemma (1.13)

$$y \mid (a-b)$$

$$\frac{n}{g} \mid (a-b) \Rightarrow a \equiv b \left(\mathrm{mod}\ \frac{n}{g}\right).$$

$\square$

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Corollary (3.12) Page 114

$p \nmid c \quad ac \equiv bc \pmod{p} \Rightarrow a \equiv b \pmod{p}$

Proof: Since $p \nmid c$ so $\gcd(p, c) = 1$.

Apply (3.11) to gives $a \equiv b \pmod{p}$

$\blacksquare$

Proposition (3.13)

Page 116.

$$a \times b \equiv 0 \pmod{n} \ \& \ \gcd(a,n)=1 \Rightarrow b \equiv 0 \pmod{n}$$

Proof: We have

$$a \times b \equiv a \times 0 \pmod{n}$$

Apply C.L. (3.11)

$xy \equiv xz \pmod{n}$ & $\gcd(x,n)=1$ then $y \equiv z \pmod{n}$

Gives

$$b \equiv 0 \pmod{n}$$

$\square$