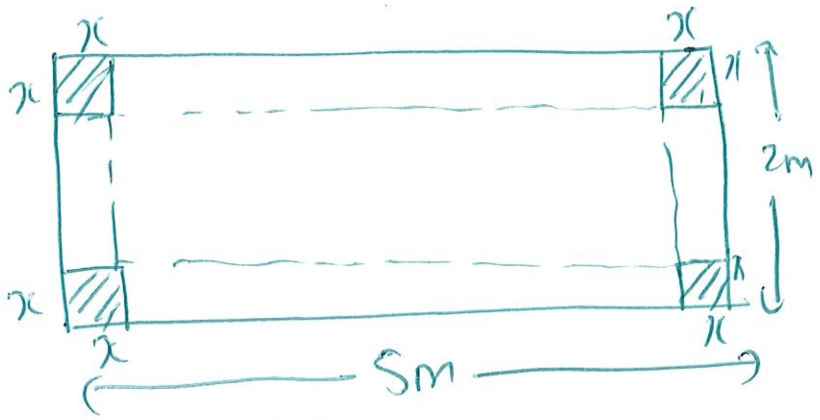
Optimization Problems



$$V = (5-2x)(2-2x)x$$
$$= (10 - 10x - 4x + 4x^2)x$$
$$= (4x^2 - 14x + 10)x$$
$$V = 4x^3 - 14x^2 + 10x$$

Diff

$$\boxed{\frac{dV}{dx} = 12x^2 - 28x + 10} = 0$$
$$6x^2 - 14x + 5 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 6,\ b = -14,\ c = 5$

$$x = \frac{14 \pm \sqrt{(-14)^2 - (4 \times 6 \times 5)}}{(2 \times 6)}$$
$$= 0.44,\ 1.89 \ X$$

$$\frac{dV}{dx} = 12x^2 - 28x + 10$$

$$\frac{d^2V}{dx^2} = 24x - 28$$

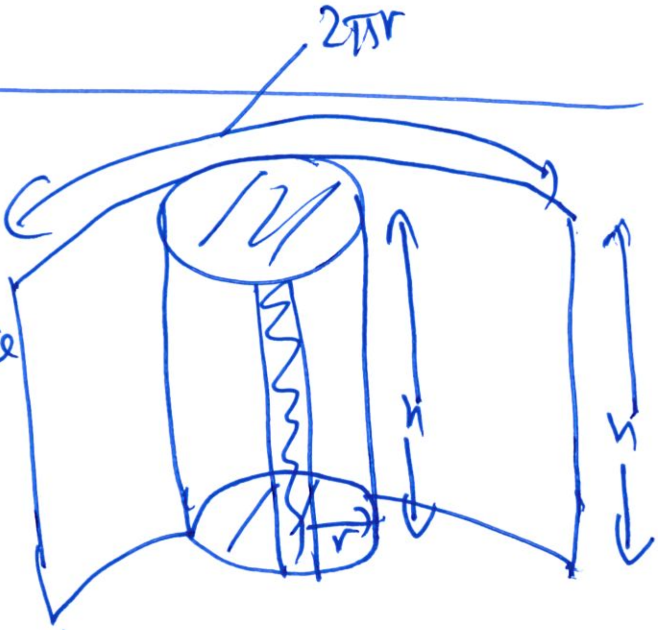
Subs $x = 0.44$ $\quad \frac{d^2V}{dx^2} = 24(0.44) - 28 < 0$

$\Rightarrow$ max.

$V = 1000 \text{ cm}^3$

Find dim so that surface area is min.

Let A = surface area.



$$A = 2\pi r^2 + 2\pi r h$$

We have $\quad \pi r^2 h = 1000 = V$

$$\boxed{h = \frac{1000}{\pi r^2}} \quad (*)$$

Hence

$$A = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\frac{1000}{\cancel{\pi} r^{\cancel{2}}}$$

$$A = 2\pi r^2 + 2000 r^{-1}$$

Diff:

$$\frac{dA}{dr} = 4\pi r + -2000 r^{-2} = 0$$

$$4\pi r = \frac{2000}{r^2}$$ 3

$$r^3 = \frac{2000}{4\pi} = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} = 5.42\,cm$$

Diff again

$$\frac{dA}{dr} = 4\pi r - 2000r^{-2}$$

$$\frac{d^2A}{dr^2} = 4\pi + 4000r^{-3}$$

$$= 4\pi + \frac{4000}{r^3}$$

$r = 5.42\,cm$

$$\frac{d^2A}{dr^2} > 0 \Rightarrow \text{min.}$$

$$h = \frac{1000}{\pi(5.42)^2} = 10.84\,cm$$

$r = 5.42\,cm$