Parametric Differentiation Sect 6F Page 311

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Ex 28 Find $\frac{dy}{dx}$ if

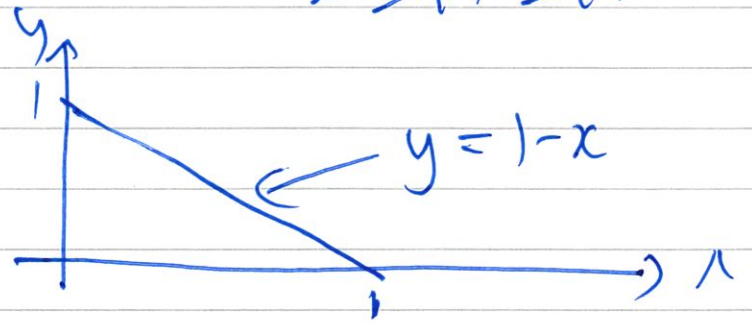
$$x = \sin^2(\theta), \quad y = \cos^2(\theta)$$

Soln:

$$\frac{dx}{d\theta} = 2\sin(\theta)\cos(\theta)$$

$$\frac{dy}{d\theta} = -2\cos(\theta)\sin(\theta)$$

$$\frac{dy}{dx} = \frac{-2\cos(\theta)\sin(\theta)}{2\sin(\theta)\cos(\theta)} = -1$$



$$\cos^2(\theta) = 1 - \sin^2(\theta)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}}$$

Ex: Fi

Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ of

$$x = r\cos(\omega t), \quad y = r\sin(\omega t)$$

Soln:

$$[\cos(kt)]' = -k\sin(kt)$$

$$\boxed{\frac{dx}{dt} = -\omega r \sin(\omega t)}$$

$$[\sin(kt)]' = k\cos(kt)$$

$$\frac{dy}{dt} = \omega r \cos(\omega t)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\omega r \cos(\omega t)}{-\omega r \sin(\omega t)}$$

$$= -\frac{\cos(\omega t)}{\sin(\omega t)} = -\cot(\omega t)$$

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left[-\cot(\omega t)\right]$$

$$= -\left[-\omega \operatorname{cosec}^2(\omega t)\right]$$

$$= \omega \operatorname{cosec}^2(\omega t)$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\omega \operatorname{cosec}^2(\omega t)}{-\omega r \sin(\omega t)}$$

$$= -\frac{1}{r}\operatorname{cosec}^2(\omega t) \cdot \underbrace{\frac{1}{\sin(\omega t)}}_{=\operatorname{cosec}(\omega t)}$$

$$= -\frac{1}{r}\operatorname{cosec}^3(\omega t)$$