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$$s = 2t^3 + t^2 - 10t$$

$$v = \frac{ds}{dt} = 6t^2 + 2t - 10$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[6t^2 + 2t - 10\right] = 12t + 2$$

$$a = \frac{d^2 s}{dt^2}$$

$$\frac{d^2 y}{dx^2}, \quad \ddot{x} = \frac{d^2 x}{dt^2}, \quad f''(x)$$

Ex: $x = A\cos(\omega t) + B\sin(\omega t)$

Show $\frac{d^2 x}{dt^2} + \omega^2 x = 0$

Soln:

$$\frac{dx}{dt} = \begin{cases} [\cos(kt)]' = -k\sin(kt) \\ [\sin(kt)]' = k\cos(kt) \end{cases}$$

$$= -\omega A \sin(\omega t) + \omega B \cos(\omega t)$$

$$\frac{d^2 x}{dt^2} = -\omega\omega A\cos(\omega t) - \omega\omega B\sin(\omega t)$$

$$= -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t)$$

$$= -\omega^2\left[A\cos(\omega t) + B\sin(\omega t)\right]$$

$$= -\omega^2 x$$

$$\frac{d^2 x}{dt^2} + \omega^2 x = -\omega^2 x + \omega^2 x = 0$$

$s = 2te^{-0.5t}$

Find t when $\dot{s} = 0 = \frac{d^2s}{dt^2}$

Soln: $s = 2te^{-0.5t}$

Product Rule $(uv)' = u'v + uv'$

Let $u = 2t \rightarrow v = e^{-0.5t}$

$u' = 2 \rightarrow v' = -0.5e^{-0.5t}$

$(e^{kt})' = ke^{kt}$

$$\frac{ds}{dt} = 2e^{-0.5t} + 2t(-0.5e^{-0.5t})$$

$$= e^{-0.5t}[2 + 2(-0.5t)]$$

$$= e^{-0.5t}[2 - t]$$

Product rule again:

Let $p = e^{-0.5t} \rightarrow q = 2 - t$

$p' = -0.5e^{-0.5t} \rightarrow q' = -1$

$$\frac{d}{dt}[e^{-0.5t}(2-t)] = -0.5e^{-0.5t}(2-t) + e^{-0.5t}(-1)$$

$$= e^{-0.5t}[1 - 0.5t + 1]$$

$$= -e^{-0.5t}[2 - 0.5t] = 0$$

$$-e^{-0.5t}[2 - 0.5t] = 0$$

$-e^{-0.5t} \neq 0$ or $2 - 0.5t = 0$

$2 = 0.5t$

$t = 4$

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$f''(x) = -\sin(x)$

$f'''(x) = -\cos(x)$

$f''''(0) = -\cos(0) = -1$