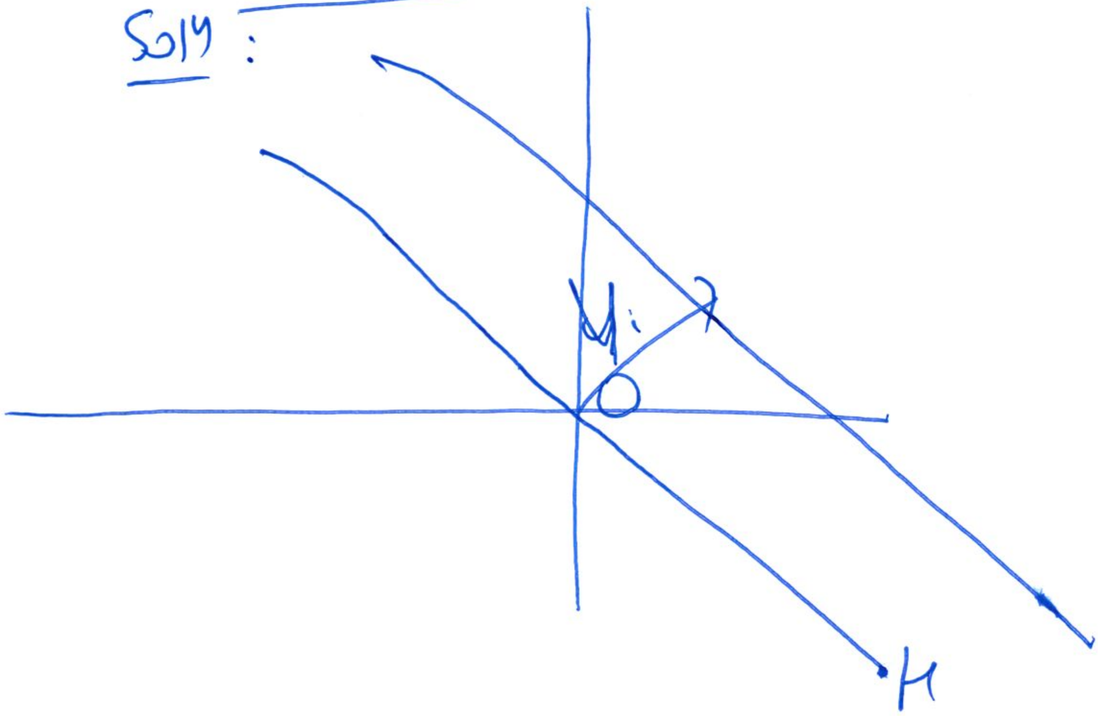
Page 746 Non-homogeneous des (1)

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

where $f(x) \neq 0$.

Solve
$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 5e^{4x}$$

Soln:



$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0.$$

$$m^2 - m - 2 = 0$$

$$(m-2)(m+1) = 0$$

$$m_1 = 2, \quad m_2 = -1$$

$$y_c = Ae^{2x} + Be^{-x}.$$

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$$Y = Ce^{4x}$$

$$y = Ae^{2x} + Be^{-x} + Ce^{4x}$$

$$Y = Ce^{4x}$$

$$\frac{dY}{dx} = 4Ce^{4x}$$

$$\frac{d^2Y}{dx^2} = 16Ce^{4x}.$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 5e^{4x}$$

$$16Ce^{4x} - 4Ce^{4x} - 2Ce^{4x} = 5e^{4x}$$

$$(16C - 4C - 2C)e^{4x} = 5e^{4x}$$

$$10C = 5$$

$$C = \frac{5}{10} = \frac{1}{2}.$$

$$Y = Ce^{4x} = \frac{1}{2}e^{4x}.$$

$$y = y_c + Y$$

$$y = Ae^{2x} + Be^{-x} + \frac{1}{2}e^{4x}$$

Determine the Particular S for

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 2x$$

Sol^n:

$$y = Ax + B$$

$$\frac{dy}{dx} = A$$

$$\frac{d^2y}{dx^2} = 0$$

$$0 - A - 2(Ax + B) = 2x$$

$$-A - 2Ax - 2B = 2x$$

Equate coeffs:

x: $-2A = 2$

$A = -1$.

$$-A - 2B = 0$$

$$1 - 2B = 0$$

$$1 = 2B \Rightarrow B = \frac{1}{2}.$$

$$y = Ax + B = -x + \frac{1}{2}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 5x^2$$

Soln:

$$Y = Ax^2 + Bx + C$$

$$\frac{dY}{dx} = 2Ax + B$$

$$\frac{d^2Y}{dx^2} = 2A$$

$$2A - 2Ax - B - 2Ax^2 - 2Bx - 2C$$
$$= 5x^2$$

$$-2Ax^2 - (2A + 2B)x + 2A - 2C \cancel{= 5x^2}$$
$$- B = 5x^2$$

Equate coeffs

x^2: $\quad -2A = 5 \Rightarrow A = \frac{-5}{2}$.

x: $\quad -2A - 2B = 0$

$$B = -A = \frac{5}{2}$$

const: $\quad 2A - 2C - B = 0$

$$2\left(\frac{-5}{2}\right) - 2C - \frac{5}{2} = 0$$

$$C = \frac{-15}{4}.$$

$$Y = Ax^2 + Bx + C = -\frac{5}{2}x^2 + \frac{5}{2}x - \frac{15}{4}.$$