Powers & Roots of Complex Nos Page 538

De - Moivre's Theorem

$$z = r\angle\theta$$

$$z^2 = r \times r \angle(\theta+\theta) = r^2 \angle 2\theta$$

$$z^3 = z^2 \times z = r^2\angle 2\theta \times r\angle\theta$$

$$= r^2 \times r \angle(2\theta+\theta)$$

$$z^3 = r^3 \angle 3\theta$$

$$z^4 = r^4 \angle 4\theta$$

$$\vdots$$

$$\boxed{z^n = r^n \angle n\theta}$$

$$\frac{1}{z^n} = z^{-n} = r^{-n} \angle(-n\theta)$$

Evaluate $(1-j)^5$.

Soln:

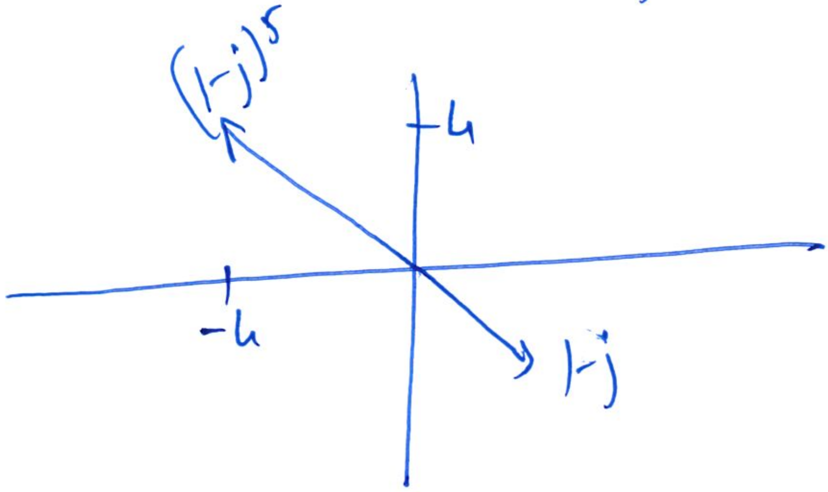
$$1-j = \sqrt{2}\angle(-45^\circ)$$

$$(1-j)^5 = \left[\sqrt{2}\angle(-45^\circ)\right]^5$$

$$= (\sqrt{2})^5 \angle(5\times(-45^\circ))$$

$$= (2^{1/2})^5 \angle -225^\circ = 2^{5/2}\angle -225^\circ$$

$= -4 + j4$



$z^n = r^n \angle n\theta$

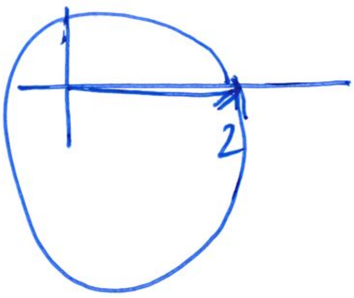
Fundamental Th of Algebra.

$(x+iy)^{1/n}$ has exactly n roots

$(2+j3)^{1/5}$ ———— 5 roots.

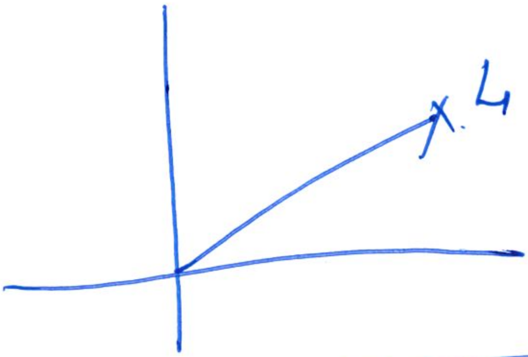
$(x-1)^{100} = 0$.

$(2+j3)^{\frac{1}{1000\,000}}$ ———— 1 000 000 roots



$2 = 2\angle 0 = 2\angle 2\pi$

$\cancel{y = e^{2x} \cos(x)}$ $= 2\angle 4\pi$



$$4\angle \pi/6 = 4\angle \frac{\pi}{6} + 2\pi$$
$$= 4\angle \frac{13\pi}{6}.$$

Solve $\quad z^2 - (2\sqrt{3} + j2) = 0$

<u>Soln</u>:

$$z^2 = 2\sqrt{3} + j2$$

$$z = \sqrt{2\sqrt{3} + j2} = \cancel{2\sqrt{}}$$

$$= (2\sqrt{3} + j2)^{1/2}.$$

$$2\sqrt{3} + j2 = 4\angle \pi/6.$$

$$(2\sqrt{3} + j2)^{1/2} = (4\angle \pi/6)^{1/2}$$

$$= 4^{1/2} \angle\left(\frac{1}{2} \times \frac{\pi}{6}\right)$$

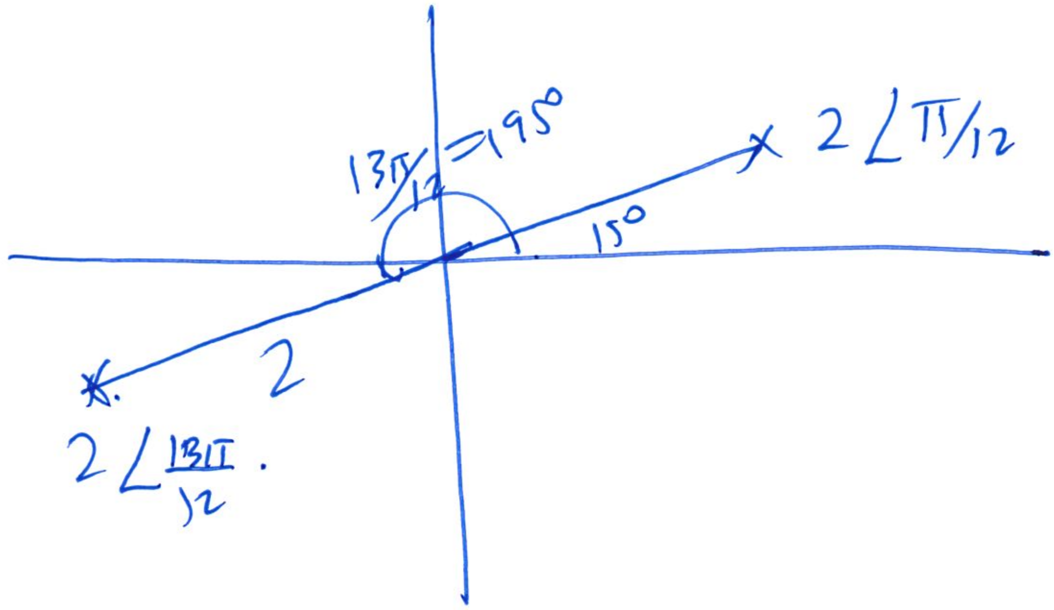
$$= 2\angle\left(\frac{\pi}{12}\right) \checkmark$$

$$4\angle\frac{\pi}{6} = 4\angle\left(\frac{\pi}{6} + 2\pi\right)$$

$$= 4\angle\frac{13\pi}{6}.$$

$$(2\sqrt{3} + j2)^{1/2} = \left(4\angle\frac{13\pi}{6}\right)^{1/2}$$

$$= 2\angle\left(\frac{13\pi}{12}\right).$$


$13\pi/12 = 195°$
$2 \angle \pi/12$
$15°$
2
$2 \angle \frac{13\pi}{12}$.