Implicit Diff Sect 6 Page 315.

Implicit
$$\begin{cases} e^{xy} + \cos^2(x) = 2 \\ \cos(xy) + \sin(x) + \ln(xy) = y \\ \sin(xy) = 1 \end{cases}$$

Explicit
$$\begin{cases} y = x^2 \\ y = \cos^2(x) + 1 \\ y = \ln(x+1) + e^{x^2} \end{cases} \quad \frac{dy}{dx}.$$

Ex: If
$$x^3 + y^2 + x + y = 2$$
find $\frac{dy}{dx}$.

Soln:
$$3x^2 + 2y\frac{dy}{dx} + 1 + \frac{dy}{dx} = 0$$
$$(2y+1)\frac{dy}{dx} = -(3x^2+1)$$
$$\frac{dy}{dx} = -\left(\frac{3x^2+1}{2y+1}\right)$$

$$x^2y^2 + 2xy = 3$$

Find $\frac{dy}{dx}$.

Soln:

$$\frac{d}{dx}(x^2y^2) = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$\frac{d}{dx}(2xy) = 2y + 2x\frac{dy}{dx}$$

We have

$$\frac{d}{dx}(x^2y^2 + 2xy = 3)$$

$$= 2xy^2 + \left(2x^2y\frac{dy}{dx}\right) + 2y + \left(2x\frac{dy}{dx}\right) = 0$$

$$(2x^2y + 2x)\frac{dy}{dx} = -(2xy^2 + 2y)$$

$$\frac{dy}{dx} = -\frac{2xy^2 + 2y}{2x^2y + 2x}$$

$$= -\frac{2y(xy+1)}{2x(xy+1)}$$

$$= -\frac{y}{x}$$

Ex $\frac{d}{dx}[\ln(y)] = \frac{1}{y}\frac{dy}{dx}$