Review of Fourier Series

The examination and test paper will ask you to find the Fourier series representation of a periodic function. It will test your ability to find the Fourier series by first calculating the Fourier coefficients.

To prepare for the test and examination you should try all the problems on chapter 17 Fourier Series which is under the Mathematical Techniques link at the http://mathsforall.co.uk/ site.

You should be able to

- recognise periodic functions and evaluate their period
- determine the Fourier coefficients of a 2π periodic function
- using the Fourier coefficients to write down the Fourier series of a 2π periodic function
- determine the algebraic and geometric properties of odd and even functions
- carry out the integration simplifications involved with odd and even functions
- determine the Fourier series of odd and even periodic functions
- determine the Fourier coefficients of a general $2L$ periodic function
- using the Fourier coefficients to write down the Fourier series of a general $2L$ periodic function
- extend the half-range series to a periodic odd, even or neither function
- determine the Fourier series of these half range series extensions
- use an appropriate value for the unknown to obtain a given series

Past Examination Questions

As I have mentioned, Fourier series was on the Mathematical Techniques 2 module whose module code is 5PAM0012. To get hold of past exam questions, you need to go to the Studynet site for Mathematical Techniques 2 and past papers are under teaching materials and these should be visible to everyone on Studynet. Just ‘search module directory’ for the module Mathematical Techniques 2 or the code

5PAM0012 and then go to teaching resources and half way down is the title 'Past Papers'.

I have added some of my own questions below. The complete solutions to these are at http://mathsforall.co.uk/ under mathematical techniques.

Sample Test and Examination Questions

1. A sinusoidal waveform is applied to a relay in a circuit. The output response is the waveform $f(t)$:

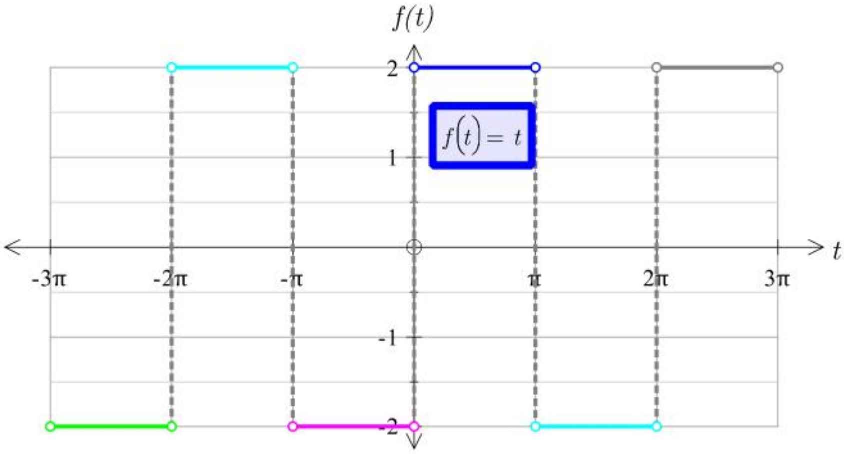

Figure 1

Obtain the Fourier series for $f(t)$.

[The general Fourier series is given by

$$f(t)=A_0+A_1\cos(t)+A_2\cos(2t)+\cdots+B_1\sin(t)+B_2\sin(2t)+\cdots$$

Where

$$A_0=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(t)\,\mathrm{d}t\,,\quad A_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\cos(kt)\,\mathrm{d}t \text{ and } B_k=\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\sin(kt)\,\mathrm{d}t]$$

2. Consider the following waveform $f(t)$:

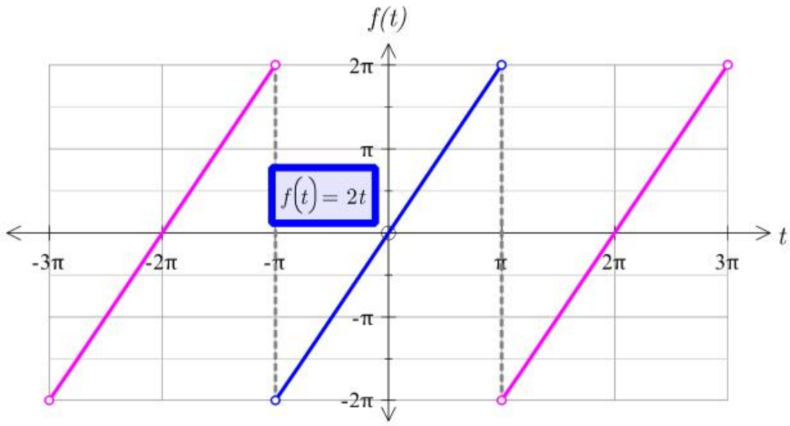

Figure 2

(i) Show that the Fourier series of this waveform $f(t)$ is given by

$$f(t) = 4\left[\sin(t) - \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} - \frac{\sin(4t)}{4} + \ldots\right]$$

(ii) By using an appropriate value for t show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

[The general Fourier series is given by

$$f(t) = A_0 + A_1\cos(t) + A_2\cos(2t) + \cdots + B_1\sin(t) + B_2\sin(2t) + \cdots$$

Where

$$A_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,\mathrm{d}t\,,\quad A_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(kt)\,\mathrm{d}t \text{ and } B_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(kt)\,\mathrm{d}t\,]$$

3. Find the Fourier sine series expansion of the function $f(x) = 1, \quad 0 < x < \pi$.

4. Determine the Fourier series to represent the periodic function shown.

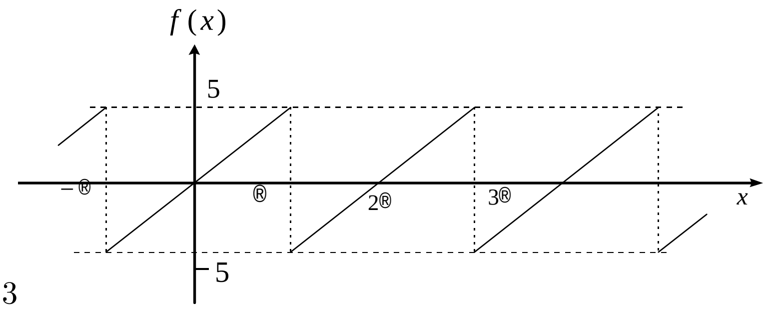

Figure 3

5. Determine the Fourier series for the periodic function shown:

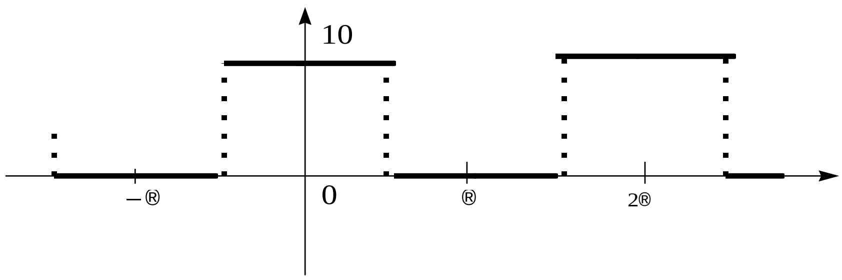

Figure 4

6. Find the Fourier series for the function defined by:

$$f(x) = 0 \qquad -® < x < 0$$
$$f(x) = x \qquad 0 < x < ®$$
$$f(x) = f(x + 2®)$$

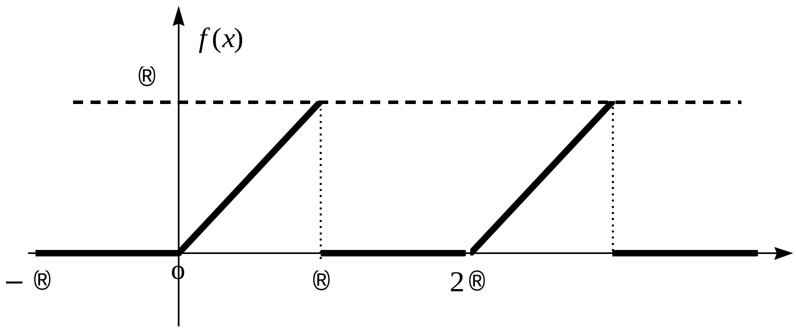

Figure 5

Brief Solutions

1. $\frac{8}{\pi}\left[\sin(t) + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \cdots\right]$

3. $\frac{4}{\pi}\left(\sin(x) + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \frac{\sin(7x)}{7} + \ldots\right)$

4. $\frac{10}{\pi}\left\{\sin(x) - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) - \frac{1}{4}\sin(4x) + \ldots.\right\}$

5. $5 + \frac{20}{\pi}\left(\cos(x) - \frac{1}{3}\cos(3x) + \frac{1}{5}\cos(5x) - \frac{1}{7}\cos(7x) + \ldots..\right)$

6. $\frac{\pi}{4} - \frac{2}{\pi}\left\{\cos(x) + \frac{1}{3^2}\cos(3x) + \frac{1}{5^2}\cos(5x) + \ldots..\right\} + \left\{\sin(x) - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) - \frac{1}{4}\sin(4x) + \ldots..\right\}$