Exercise **1(a)**

1. You have a rectangular sheet of metal of dimensions 60 inches by 84 inches. You want to cut this metal into smaller identical squares. What is the largest size square you would need in order to ensure there is no metal left over?

2. Find $\gcd(57, \;\; 209)$. Hence or otherwise simplify the fraction $\frac{57}{209}$.

3. Determine $\gcd(65, \;\; 1001)$. Hence write the ratio $65:1001$ in its simplest form.

4. In music the fundamental frequency f_0 is the gcd of the frequencies f_n of the harmonics. Find the fundamental frequency f_0 of the harmonics
$$f_1 = 200\text{Hz}, \;\; f_2 = 300\text{Hz}, \;\; f_3 = 400\text{Hz} \;\text{ and }\; f_4 = 500\text{Hz}$$

5. Find $^{2}(n)$ (tau function) for the following n values:
(a) 10 (b) 100 (c) 120 (d) 101

6. Determine the gcd of the following integers:
(a) $-12, \; 34$ (b) $-36, \; -60$ (c) $60, \; -72$ (d) $1001, \; 182$

7. Plot the graph $24x + 120y = \gcd(24, \;\; 120)$. By using this graph or otherwise find two integer solutions to this equation $24x + 120y = \gcd(24, \;\; 120)$.

8. Determine a particular integer solution to $56x + 60y = \gcd(56, \;\; 60)$.

9. Determine $\gcd(66, \;\; 165, \;\; 253)$.

10. Determine $\gcd(a, \;\; a^2)$ where a is a non-zero integer.

11. Determine $\gcd(a+b, \;\; a^2 - b^2)$ where integers $a+b$ and $a^2 - b^2$ are not both zero.

12. Prove or disprove the following statement:

$$a \mid b \text{ and } c \mid d \text{ implies } (a+c) \mid (b+d)$$

13. (a) Find the possible values of the integer a such that $a \mid 0$.

(b) Find the possible values of the integer a such that $a \mid 2$.

14. Prove that $a \mid b \Leftrightarrow ac \mid bc$ provided $c \neq 0$.

15. Let $a \mid (b+c)$ and $a \mid b$. Show that $a \mid c$.

16. Prove that if $a \mid b$ and $a \mid c$ then $a \mid (b^2 - c^2)$.

17. Show that $a \mid (b \times c) \not\Rightarrow a \mid b$ or $a \mid c$. *Under what circumstances do you think this result is true; that is $a \mid (b \times c) \Rightarrow a \mid b$ or $a \mid c$?

18. Prove that $\gcd(-a, -b) = \gcd(a, b)$ where a and b are not both zero.

19. Prove that if $a \mid b_1,\ a \mid b_2,\ a \mid b_3, \cdots$ and $a \mid b_n$ then $a \mid (b_1x_1 + b_2x_2 + \cdots + b_nx_n)$ for any integers $x_1,\ x_2,\ \cdots,\ x_n$.

20. Prove that $\gcd(a, b, c) = \gcd(a, \gcd(b, c))$.

Brief Solutions

1. 12
2. 19, $\frac{3}{11}$
3. 13, 5:77
4. 100Hz
5. (a) 4 (b) 9 (c) 16 (d) 2
6. (a) 2 (b) 12 (c) 12 (d) 91
7. $x = -4, y = 1$ or $x = 6, y = -1$
8. $x = -1, y = 1$
9. 11

10. $|a|$

11. $|a+b|$ 12. False.

13. (a) Any integer (b) $\pm 1, \pm 2$.

17. $22 \mid 11\times 12$ but $22 \nmid 11$ or $22 \nmid 12$.