The minor of 5 is $= \det \begin{pmatrix} -1 & 3 \\ -4 & -9 \end{pmatrix}$

$= 9 + 12 = 21$

Cofactor of 5 is -21 =

Cofactor of $a_{13} = (-1)^{1+3} M_{13} = M_{13}$

$a \det \begin{pmatrix} e & f \\ h & i \end{pmatrix} - b \det \begin{pmatrix} d & f \\ g & i \end{pmatrix} + c \det \begin{pmatrix} d & e \\ g & h \end{pmatrix}$

$\det A = -2 \det \begin{pmatrix} -6 & 6 \\ 3 & -7 \end{pmatrix} - 0 + 1 \det \begin{pmatrix} -1 & 5 \\ -6 & 6 \end{pmatrix}$

$= -2 [42 - 18] + [-6 + 30] = -24.$

$|\det(A)| = 24.$

cofactor of $1 = \det\begin{pmatrix} 9 & 7 \\ 1 & 0 \end{pmatrix} = -7$

______ $-1 = -\det\begin{pmatrix} 3 & 7 \\ -2 & 0 \end{pmatrix} = -[0+14] = -14.$

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adjoint of $A = C^T = \begin{pmatrix} -7 & -14 & 21 \\ 5 & 10 & 1 \\ -52 & 8 & 12 \end{pmatrix}^T$

$$= \begin{pmatrix} -7 & 5 & -52 \\ -14 & 10 & 8 \\ 21 & 1 & 12 \end{pmatrix}$$

<u>Proof:</u>

$$(A \operatorname{adj} A)_{ij} = a_{i1} C_{j1} + a_{i2} C_{j2} + \cdots + a_{in} C_{jn}$$

$$(A \operatorname{adj}(A)) = \begin{pmatrix} \det(A) & 0 & 0 & \cdots & 0 \\ & \det(A) & & 0 & \\ & 0 & \ddots & & \\ & & & & \det(A) \end{pmatrix}$$

$$= \det(A) \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

$$A \operatorname{adj}(A) = \det(A)\, I$$

$$A \, adj(A) = det(A) \, I$$

Proof: We have from previous proposition:

$$A \, adj(A) = det(A) . I$$

$$\frac{1}{det(A)} A . adj(A) = I$$

$$A . \underbrace{\frac{1}{det(A)} adj(A)}_{= A^{-1}} = I$$

$$\boxed{A^{-1} = \frac{1}{det A} adj(A).}$$

$$det \begin{pmatrix} 1 & -1 & 5 \\ 3 & 9 & 7 \\ -2 & 1 & 0 \end{pmatrix} = -2 \, det \begin{pmatrix} -1 & 5 \\ 9 & 7 \end{pmatrix} - 1 \, det \begin{pmatrix} 1 & 5 \\ 3 & 7 \end{pmatrix} + 0$$

$$= -2 [-7 - 45] - [7 - 15]$$

$$= 112$$

$$A^{-1} =$$