Solution by Alia Razaq, Amy Washington and Sally Wright

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\sin(x)-\cos(x)}dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin(x)(\sin x+\cos x)}{\sin^2(x)-\cos^2(x)}dx$$ times by sin(x)+cos(x)

$$=-\left[\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\cos 2x}dx+\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{\cos 2x}dx\right]$$

$$=-\left[\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{\cos 2x}dx+\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos 2x}dx\right]$$

$$=-\frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{\cos 2x}dx+\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos 2x}dx\right]$$

$$=\frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \frac{\cos 2x-1}{\cos 2x}dx-\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos 2x}dx\right]$$

$$=\frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \frac{\cos 2x}{\cos 2x}dx-\int_0^{\frac{\pi}{2}} \frac{1}{\cos 2x}dx-\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos 2x}dx\right]$$

$$=\frac{1}{2}\left[\int_0^{\frac{\pi}{2}} 1dx-\int_0^{\frac{\pi}{2}} \frac{1}{\cos 2x}dx-\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\cos 2x}dx\right]$$

$$=\frac{1}{2}\left[\int_0^{\frac{\pi}{2}} 1dx-\int_0^{\frac{\pi}{2}} \sec 2x dx-\int_0^{\frac{\pi}{2}} \tan 2x dx\right]$$

$$=\frac{1}{2}\left[x-\frac{1}{2}\ln|\sec 2x+\tan 2x|-\frac{1}{2}\ln|\sec 2x|\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{2}\left[\frac{\pi}{2}-\frac{1}{2}\ln|-1+0|-\frac{1}{2}\ln|-1|\right]-\frac{1}{2}\left[0-\frac{1}{2}\ln|1+0|-\frac{1}{2}\ln|1|\right]$$

$$\ln|-1|=\ln|1|=0$$

therefore :

$$=\left(\frac{1}{2}\left(\frac{\pi}{2}\right)\right)-0$$

$$=\frac{\pi}{4}$$