Mathematics,
Bonus Question, Series

December 3, 2012

It's not very often that mathematics features in popular culture, but in 1879 Gilbert and Sullivan released a comic opera titled *The Pirates of Penzance* in which there is a song called, *I am the Very Model of a Modern Major General.* In this song mathematics has several lines dedicated to it, look it up, you'll no doubt recognise the tune!

...I'm very well acquainted, too, with matters mathematical,
I understand equations, both the simple and quadratical,
About binomial theorem I'm teeming with a lot o' news,
With many cheerful facts about the square of the hypotenuse.
I'm very good at integral and differential calculus...

Now then, you, much like the aforementioned major general are well acquainted with matters mathematical and seeing as the song mentions Binomial Theorem. The binomial theorem allows us to expand a power into a sum. Other methods for expanding functions into sums exist, another method is the Maclaurin Series, which is the Taylor series around the origin. Power series (such as a Maclaurin series) have several intertesting and useful properties but we will make use of just one, and that is:

"One series may be substituted in another; provided that the values of the series substituted are in the interval of convergence of the other series".

Although the final part of the statement may seem confusing we can just take it to be true for now. Finally, the problem I would like you to tackle:

$$\int_0^1 e^{sin(x)} dx$$

Your first reaction may be to try a substitution, which you may try, but you won't get far with it, instead we're going to use the Maclaurin series, which is defined as:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + ...$$

Questions. Let's agree to go up to third term in the series.

1. Find the Maclaurin series for e^x
2. Find the Maclaurin series for $sin(x)$

   Now, we make use of the property stated earlier.

3. Substitute in the series of $sin(x)$ into the series for e^x, only substitute in the first two terms of the $sin(x)$ series or things may start to get messy. *hint: replace the x in your e^x series with your series for $sin(x)$*
4. perform the integral stated earlier by integrating term by term

   If you were to ask a computer algebra system to evaluate this integral you would get an answer of ≈ 1.63

5. Did you get this? If not, can you think of any reasons why?