Exercise 5(c)

Workbook questions in bold.

1. Find the number N_0 and the least positive integer n such that $\forall n > N_0$ we have the inequality
$$\left|\frac{1}{2n}\right| < \varepsilon$$
for
(a) $\varepsilon = 0.1$ (b) $\varepsilon = 0.01$ (c) $\varepsilon = 1\times 10^{-3}$
(d) $\varepsilon = 1\times 10^{-6}$
Check your results.

2. Find the number N_0 and the least positive integer n such that $\forall n > N_0$ we have the inequality
$$\left|\frac{2n+1}{n+1} - 2\right| < \varepsilon$$
for
(a) $\varepsilon = 0.1$ (b) $\varepsilon = 0.01$ (c) $\varepsilon = 1\times 10^{-3}$
(d) $\varepsilon = 1\times 10^{-6}$

3. By using the formal definition of the limit of the sequence prove the following:
(a) $\lim\limits_{n\to\infty}\left(\frac{1}{n+1}\right) = 0$ (b) $\lim\limits_{n\to\infty}\left(\frac{1}{n^2+1}\right) = 0$ (c) $\lim\limits_{n\to\infty}\left(\frac{1}{n^3+1}\right) = 0$

4. **Prove that** $\lim\limits_{n\to\infty}\left(\frac{c}{n}\right) = 0$ **for any** $c \in \mathbb{R}$ **.**

5. By using the formal definition of the limit of the sequence prove the following:
(a) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right) = 1$ (b) $\lim\limits_{n\to\infty}\left(1-\frac{1}{n}\right) = 1$ (c) $\lim\limits_{n\to\infty}\left(9+\frac{1}{n}\right) = 9$
(d) $\lim\limits_{n\to\infty}\left(k+\frac{1}{n}\right) = k$ where k is a real number

6. By using the formal definition of the limit of the sequence prove the following:
(a) $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n}}\right) = 0$
(b) $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt[k]{n}}\right) = 0$ where $k \in \mathbb{N}$

7. Prove the following limits:

(a) $\lim_{n\to\infty}\left(\frac{n^2-1}{n^2+1}\right)=1$ (b) $\lim_{n\to\infty}\left(\frac{n^2-1}{2^n}\right)=0$

8. Prove that $\lim_{n\to\infty}\left[\frac{\cos(n)}{n}\right]=0$ using the formal definition of a limit of sequence.

9. (i) Prove the following $\lim_{n\to\infty}\left(\frac{1}{2^n}\right)=0$

(ii) Show that if $|x|>1$ then $\lim_{n\to\infty}\left(\frac{1}{x^n}\right)=0$

(iii) Hence, or otherwise, prove that $\lim_{n\to\infty}\left(e^{-n}\right)=0$

10. By using the formal definition of the limit of the sequence prove that

$$\lim_{n\to\infty}\left(\frac{1}{n^n}\right)=0$$

Solutions 5(c)

1. $N_0=\frac{1}{2\varepsilon}$.

(a) $n=6$ (b) $n=51$ (c) $n=501$. (d) $n=500\,001$