Exercise **3(e)**

Workbook questions in bold, **Questions 4 and 5.**

1. Let A be any set and I_A be the identity function on the set A.

(i) Prove that I_A is bijective.

(ii) Determine I_A^{-1}

2. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=(x-1)^3+2$$

(i) Show that the given function f is bijective.

(ii) Determine the formula for $f^{-1}(x)$.

(iii) Determine $\left(f\circ f^{-1}\right)(x)$ and $\left(f^{-1}\circ f\right)(x)$.

3. Let $A=\{1,\ 2,\ 3\}$ and $B=\{a,\ b,\ c\}$. Let $f:A\to B$ be defined by

$$f(1)=b,\quad f(2)=c,\quad f(3)=a$$

(i) Show that the function f is bijective.

(ii) Specify f^{-1}.

(iii) Find $f\circ f^{-1}$ and $f^{-1}\circ f$.

4. Let $f:A\to B$ be a bijective function. Prove that

$$f\circ f^{-1}=I_B$$

5. Let $g:A\to B$ and $f:B\to C$ are injective functions. Prove that $f\circ g:A\to C$ is also injective.

6. Let $g:A\to B$ and $f:B\to C$ are surjective functions. Prove that $f\circ g:A\to C$ is also surjective.

7. Let $g:A\to B$ and $f:B\to C$ be bijective functions. Prove that

$$\left(f\circ g\right)^{-1}=g^{-1}\circ f^{-1} \quad \text{and} \quad \left(g\circ f\right)^{-1}=f^{-1}\circ g^{-1}$$

8. Let $f:A\to B$ be a function. Prove that there exists a function $g:B\to A$ such that

$$g\circ f=I_A \quad \Leftrightarrow \quad f \text{ is an injection}$$

The function g is called the **left inverse** of f .

9. Let $f:A\to B$ be a function. Prove that there exists a function $g:B\to A$ such that

$$f\circ g=I_B \quad \Leftrightarrow \quad f \text{ is an surjection}$$

The function g is called the **right inverse** of f .