Exercise 2b

1. Show that the following series are convergent and determine their sum:

   (a) $\sum_{k=1}^{\infty}\left[\frac{1}{(2k-1)(2k+1)}\right]$

   (b) $\sum_{k=1}^{\infty}\left[\frac{1}{(k+1)(k+3)}\right]$

   (c) $\sum_{k=1}^{\infty}\left[\frac{1}{(4k+1)(4k-3)}\right]$

2. Show that $\sum_{k=3}^{\infty}\left(\frac{1}{k^2-4}\right)=\frac{25}{48}$

3. Determine whether the following series converges:

   $$\sum_{k=1}^{\infty}\left(\frac{1}{k(k+1)(k+2)}\right)$$

   If the series converges then determine its sum.

4. Prove that $\sum_{k=1}^{\infty}\left(\frac{1}{2k}\right)$ is divergent.

5. (i) Show that

   $$\sum_{k=m+1}^{\infty}\left(\frac{1}{k^2-m^2}\right)=\frac{1}{2m}\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{2m}\right) \quad \text{where } m\in\mathbb{N}$$

   (ii) Hence, or otherwise, determine

   (a) $\sum_{k=4}^{\infty}\left(\frac{1}{k^2-9}\right)$ (b) $\sum_{k=5}^{\infty}\left(\frac{1}{k^2-16}\right)$ (c) $\sum_{k=6}^{\infty}\left(\frac{1}{k^2-25}\right)$

6. Prove that $\sum_{k=1}^{\infty}\left(\frac{1}{\sqrt{k}}\right)$ diverges.

Solutions

1. (a) $\frac{1}{2}$ (b) $\frac{5}{12}$ (c) $\frac{1}{4}$

3. $\frac{1}{4}$

5. (ii) (a) $\frac{49}{120}$ (b) $\frac{761}{2240}$ (c) $\frac{7381}{25200}$