Exercise **1(h)**

1. Prove that for all natural numbers *n*, 9 divides $10^n - 1$.

2. Prove that for all natural numbers *n*,
$$3 \mid \left(n^3 - n\right).$$

3. Show that for every natural number n
$$3 \mid n(n+1)(n+2)$$

4. Prove that for all natural numbers *n*
$$n^2 - n \text{ is an even number}$$

5. Prove that for all natural numbers n
$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a\left(1 - r^n\right)}{1 - r} \qquad [\, r \neq 1 \,]$$
where a and r are real numbers.
[This is a geometric series with first term equal to a and common ratio r]

6. Prove that for all natural numbers *n* we have the following trigonometric identity:
$$\sin(x) + \sin(2x) + \ldots + \sin(nx) = \frac{\cos\left(\frac{x}{2}\right) - \cos\left(\frac{2n+1}{2}x\right)}{2\sin\left(\frac{x}{2}\right)}$$
where *x* is a real number such that $\sin\left(\frac{x}{2}\right) \neq 0$.

7. Prove the binomial theorem for the natural number n:
If a and b are real numbers then the binomial theorem says that for all natural numbers *n*
$$\left(a+b\right)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots + b^n$$