Exercise **1(g)**

1. Show that for all natural numbers, n
$$2+4+6+...+2n=n(n+1)$$

2. Prove that for all natural numbers, n
$$2+5+8+...+(3n-1)=\frac{1}{2}n(3n+1)$$

3. Prove that for all natural numbers, n
$$1^3+2^3+3^3+...+n^3=\frac{1}{4}n^2(n+1)^2$$

4. Prove that for all natural numbers, n
$$1^3+2^3+3^3+...+n^3=(1+2+3+4+...+n)^2$$
[Hint: Use the result of Question 3].

5. Prove that for all natural numbers, n
$$(1\times 2)+(2\times 3)+(3\times 4)+...+n(n+1)=\frac{1}{3}n(n+1)(n+2)$$

6. Prove that for all natural numbers, n
$$(1\times 2\times 3)+(2\times 3\times 4)+(3\times 4\times 5)+...+n(n+1)(n+2)=\frac{1}{4}n(n+1)(n+2)(n+3)$$

7. Show that for every natural number, n
$$1+r+r^2+...+r^n=\frac{1-r^{n+1}}{1-r} \qquad (r\neq 1)$$
[This is the geometric progression with the first term =1]

8. Show that for all natural numbers, n
$$1+2+2^2+...+2^n=2^{n+1}-1$$

9. Prove that for all natural numbers, n
$$1^3+3^3+5^3...+(2n-1)^3=n^2(2n^2-1)$$

10. Prove that for all natural numbers, n
$$1^4+2^4+3^4+...+n^4=\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

11. Prove that for all natural numbers, n

$$1^5+2^5+3^5+...+n^5=\frac{n^2\left(n+1\right)^2\left(2n^2+2n-1\right)}{12}$$