Supplementary Questions on Differentiation

1. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in each of the following cases:

   (a) $y = \left(1 + \sqrt[3]{x}\right)^3$

   (b) $y = a\tan\left(\dfrac{x}{k} + b\right)$

   (c) $y = \log_{10}\left(x - \cos\left(x\right)\right)$

   (d) $y = \sin\left(x\right).e^{\cos\left(x\right)}$

   (e) $y = e^{-x^2}\ln\left(x\right)$

   (f) $y = x\tan^{-1}\left(\sqrt{x}\right)$

   (g) $y = \sqrt[3]{1 + x\sqrt{x+3}}$

   (h) $y = \sin^{-1}\left(x\right) + \sqrt{1-x^2}$

   (i) $y = 2^{\frac{x}{\ln\left(x\right)}}$

   (j) $y = \left[\tan\left(2x\right)\right]^{\cot\left(\frac{x}{2}\right)}$

   (k) $y = \cos^{-1}\left(\dfrac{x^{2n}-1}{x^{2n}+1}\right)$

   (l) $y = \tan^{-1}\left(\tanh\left(x\right)\right)$

   (m) $y = \cosh\left(\sinh\left(x\right)\right)$

   (n) $y = \sqrt[4]{\left(1 + \tanh^2\left(x\right)\right)^3}$

2. Show that (a) if $y = \ln\left(\dfrac{1}{1+x}\right)$ then $xy' + 1 = e^y$;

   (b) if $y = \dfrac{\sin^{-1}\left(x\right)}{\sqrt{1-x^2}}$ then $\left(1-x^2\right)y' - xy = 1$.

3. What is the slope of the tangent to the ellipse

   $$\frac{x^2}{2} + \frac{y^2}{4} = 1 \text{ at the point } \left(1, \sqrt{2}\right)?$$

4. What is the slope of the tangent to the circle

   $$\left(x-1\right)^2 + \left(y+3\right)^2 = 17 \text{ at the point } \left(2, 1\right)?$$

5. Verify that

   (a) if $x = \dfrac{1+t}{t^3}$, $y = \dfrac{3}{2t^2} + \dfrac{2}{t}$ then $xy'^3 = 1 + y'$ where $y' = \dfrac{\mathrm{d}y}{\mathrm{d}x}$;

(b) if $x=\cosh\left(2t\right),\ y=\sinh\left(2t\right)$ then $yy'=x$ where $y'=\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Brief solutions

1. (a) $1+\dfrac{2}{\sqrt[3]{x}}+\dfrac{1}{\left(\sqrt[3]{x}\right)^2}$ (b) $\dfrac{a}{k}\left[1+\tan^2\left(\dfrac{x}{k}+b\right)\right]$ (c) $\dfrac{1}{\ln\left(10\right)}\left[\dfrac{1+\sin\left(x\right)}{x-\cos\left(x\right)}\right]$

(d) $e^{\cos\left(x\right)}\left[\cos^2\left(x\right)+\cos\left(x\right)-1\right]$ (e) $\dfrac{e^{-x^2}}{x}\left[1-2x^2\ln\left(x\right)\right]$

(f) $\tan^{-1}\left(\sqrt{x}\right)+\dfrac{\sqrt{x}}{2\left[1+x\right]}$ (g) $\dfrac{3x^2+6x}{6\left(1+x\left(x+3\right)^{\frac{1}{2}}\right)^{\frac{2}{3}}\sqrt{\left(x^3+3x^2\right)}}$

(h) $\sqrt{\dfrac{1-x}{1+x}}$ (i) $2^{\frac{x}{\ln\left(x\right)}}\dfrac{\ln\left(2\right)}{\ln^2\left(x\right)}\left[\ln\left(x\right)-1\right]$

(j) $\left[\tan\left(2x\right)\right]^{\cot\left(\frac{x}{2}\right)}\left[2\cot\left(\dfrac{x}{2}\right)\dfrac{\sec^2\left(2x\right)}{\tan\left(2x\right)}-\dfrac{1}{2}\operatorname{cosec}^2\left(\dfrac{x}{2}\right)\ln\left[\tan\left(2x\right)\right]\right]$

(k) $-\dfrac{2nx^{n-1}}{x^{2n}+1}$ (l) $\dfrac{1-\tanh^2\left(x\right)}{1+\tanh^2\left(x\right)}$

(m) $\sinh\left(\sinh\left(x\right)\right)\cosh\left(x\right)$ (n) $\dfrac{3\tanh\left(x\right)\left(1-\tanh^2\left(x\right)\right)}{2\left(1+\tanh^2\left(x\right)\right)^{\frac{1}{4}}}$

3. $-2^{\frac{3}{2}}$
4. $-\dfrac{1}{4}$