

Supplementary Exercises on Indefinite Integration

1. Determine each of the following indefinite integrals by dividing numerator of the rational function by its denominator and separating the quotient in this division:

$$(a) \int \frac{x}{x+4} dx$$

$$(b) \int \frac{x}{2x+1} dx$$

$$(c) \int \frac{x^2-1}{x^2+1} dx$$

$$(d) \int \frac{(1+x)^2}{x^2+1} dx$$

$$*(e) \int \frac{Ax}{a+bx} dx$$

$$(f) \int \frac{x^4}{x^2+1} dx$$

2. Find the following integrals by separating a complete square:

$$(a) \int \frac{1}{x(x-1)} dx$$

$$(b) \int \frac{1}{x^2-7x+10} dx$$

$$(c) \int \frac{1}{2-3x^2} dx$$

$$(d) \int \frac{1}{4x^2+4x+5} dx$$

$$(e) \int \frac{1}{\sqrt{4x-3-x^2}} dx$$

$$(f) \int \frac{1}{\sqrt{2-6x-9x^2}} dx$$

3. Determine the following integrals by using trigonometric formulae to reduce the integrands:

$$(a) \int \frac{1}{1-\cos(x)} dx$$

$$(b) \int \frac{1}{1+\sin(x)} dx$$

$$(c) \int \frac{1-\cos(x)}{1+\cos(x)} dx$$

$$(d) \int [\tan^2(x) + \tan^4(x)] dx$$

$$(e) \int \frac{\cos(2x)}{1+\sin(x)\cos(x)} dx$$

$$(f) \int \sin(2x)\sin(5x) dx$$

$$(g) \int \cos(x)\cos(2x)\cos(3x) dx$$

$$(h) \int \frac{\sin^3(x)}{\sqrt{\cos(x)}} dx$$

$$(i) \int \frac{1}{\cos^4(x)} dx$$

$$(j) \int \tan^4(x) dx$$

$$(k) \int \sin^5(x) dx$$

$$*(l) \int \frac{1}{\sin^6(x)} dx$$

4. Determine the following by using integration by parts:

(a) $\int x \sin(2x) \, dx$

(b) $\int 3^x x \, dx$

(c) $\int x \tan^{-1}(x) \, dx$

*(d) $\int \cos^{-1}(x) \, dx$

(e) $\int \frac{\log_{10}(x)}{x^3} \, dx$

(f) $\int \frac{\sin^{-1}(\sqrt{x})}{\sqrt{1-x}} \, dx$

(g) $\int x^2 \ln(1+x) \, dx$

(h) $\int \cos(\ln(x)) \, dx$

*(i) $\int \sqrt{a^2 + x^2} \, dx$

(j) $\int \frac{x^2}{\sqrt{1-x^2}} \, dx$

5. Determine the following integrals by method of substitution:

(a) $\int \frac{1}{1 + \sqrt{x+1}} \, dx$

(b) $\int \frac{x^3}{\sqrt{x-1}} \, dx$

(c) $\int \frac{\sqrt{x}}{\sqrt{x} - \sqrt[3]{x}} \, dx \quad (u = \sqrt[6]{x})$

(d) $\int \frac{e^{2x}}{\sqrt[4]{e^x + 1}} \, dx$

(e) $\int \frac{\ln(\tan(x))}{\sin(x) \cos(x)} \, dx$

*(f) $\int \frac{\sqrt{1 + \ln(x)}}{x \ln(x)} \, dx$

(g) * $\int \frac{1}{x^2 \sqrt{x^2 + a^2}} \, dx$

6. Determine the following integrals by expressing them into partial fractions:

(a) $\int \frac{x^2 + 1}{x(x^2 - 1)} \, dx$

(b) $\int \frac{2}{(x-1)(x-2)(x-3)} \, dx$

(c) $\int \frac{x^5 - x^3 - x^2}{x^2 - 1} \, dx$

(d) $\int \frac{4x^3}{(x^2 + 1)^2} \, dx$

(e) $\int \frac{x^6 - x^2 + 1}{(x-1)^3} \, dx$

(f) $\int \frac{3x^2 + 3}{x^3 - 3x - 2} \, dx$

Brief Solutions

1. (a) $x - 4 \ln|x + 4| + C$ (b) $\frac{1}{4} [2x - \ln|2x + 1|] + C$ (c) $x - 2 \tan^{-1}(x) + C$

(d) $x + \ln|x^2 + 1| + C$ (e) $\frac{A}{b^2} [bx - a \ln|a + bx|] + C$

(f) $\frac{x^3}{3} - x + \tan^{-1}(x) + C$

2. (a) $C - \ln \left| \frac{x}{1-x} \right|$ (b) $C - \frac{1}{3} \ln \left| \frac{x-2}{5-x} \right|$ (c) $\frac{1}{\sqrt{6}} \tanh^{-1} \left(\frac{\sqrt{3x}}{\sqrt{2}} \right) + C$

(d) $\frac{1}{4} \tan^{-1} \left(x + \frac{1}{2} \right) + C$ (e) $\sin^{-1}(x - 2) + C$ (f) $\frac{1}{3} \sin^{-1} \left(\frac{(3x + 1)}{\sqrt{3}} \right) + C$

3. (a) $C - \frac{1}{\tan \left(\frac{x}{2} \right)}$ (b) $C - \frac{2}{\tan \left(\frac{x}{2} \right) + 1}$ (c) $2 \tan \left(\frac{x}{2} \right) - x + C$

(d) $\frac{\tan^3(x)}{3} + C$ (e) $\ln |2 + \sin(2x)| + C$

(f) $\frac{1}{2} \left[\frac{\sin(3x)}{3} - \frac{\sin(7x)}{7} \right] + C$ (g) $\frac{1}{4} \left[\frac{\sin(6x)}{6} + \frac{\sin(4x)}{4} + x + \frac{\sin(2x)}{2} \right] + C$

(h) $2\sqrt{\cos(x)} \left[\frac{\cos^5(x)}{5} - 1 \right] + C$ (i) $\tan(x) + \frac{\tan^3(x)}{3} + C$

(j) $\frac{\tan^3(x)}{3} - \tan(x) + x + C$ (k) $C - \cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5}$

(l) $C - \left[\cot(x) + \frac{2\cot^3(x)}{3} + \frac{\cot^5(x)}{5} \right]$

4. (a) $\frac{1}{4} [\sin(2x) - 2x \cos(2x)] + C$ (b) $\frac{1}{\ln^2(3)} 3^x [x \ln(3) - 1] + C$

(c) $\frac{1}{2} [x^2 \tan^{-1}(x) - x + \tan^{-1}(x)] + C$ (d) $x \cos^{-1}(x) - \sqrt{1 - x^2} + C$

(e) $C - \frac{1}{4x^2 \ln(10)} [\ln(x^2) + 1]$ (f) $2 [\sqrt{x} - \sqrt{1 - x} \sin^{-1}(\sqrt{x})] + C$

(g) $\frac{x^3}{9} [3 \ln(1 + x) - 1] + \frac{1}{3} \left[\frac{x^2}{2} - x + \ln|1 + x| \right] + C$

(h) $\frac{1}{2} [x \cos(\ln(x)) + x \sin(\ln(x))] + C$ (i) $\frac{1}{2} \left[a^2 \sinh^{-1} \left(\frac{x}{a} \right) + x \sqrt{a^2 + x^2} \right] + C$

(j) $\frac{1}{2} [\sin^{-1}(x) - x \sqrt{1 - x^2}] + C$

5. (a) $2 [\sqrt{x + 1} - \ln|1 + \sqrt{x + 1}|] + C$

(b) $\frac{2}{35} (x - 1)^{\frac{1}{2}} [5(x - 1)^3 + 21(x - 1)^2 + 35(x - 1) + 35] + C$

(c) $x + \frac{6}{5} x^{\frac{5}{6}} + \frac{3}{2} x^{\frac{2}{3}} + 2x^{\frac{1}{2}} + 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} + 6 \ln \left| x^{\frac{1}{6}} - 1 \right| + C$

$$(d) \frac{4}{21}(e^x + 1)^{\frac{3}{4}}[3e^x - 4] + C$$

$$(e) \frac{1}{2}\ln^2(\tan(x)) + C$$

$$(f) 2\sqrt{1 + \ln(x)} - \ln \left| \frac{1 + \sqrt{1 + \ln(x)}}{1 - \sqrt{1 + \ln(x)}} \right| + C$$

$$(g) C - \frac{2}{a^2 \left[e^{2 \sinh^{-1}\left(\frac{x}{a}\right)} - 1 \right]}$$

$$6. (a) \ln \left| \frac{x^2 - 1}{x} \right| + C$$

$$(b) \ln \left| \frac{x^2 - 4x + 3}{x^2 - 4x + 4} \right| + C$$

$$(c) \frac{1}{4} \left(x^4 - 4x - 2 \ln \left| \frac{x-1}{x+1} \right| \right) + C$$

$$(d) 2 \left[\ln |x^2 + 1| + \frac{1}{x^2 + 1} \right] + C$$

$$(e) \frac{x^4}{4} + x^3 + 3x^2 + 10x + 14 \ln |x - 1| - \frac{4}{x - 1} - \frac{1}{2(x - 1)^2} + C$$

$$(f) \frac{1}{3} \left[4 \ln |x + 1| + \frac{6}{x + 1} + 5 \ln |x - 2| \right] + C$$