Sample Examination Questions for Engineering Mathematics

Chapter 6

1. Differentiate each of the following with respect to the variable in each case.

(i) $y = 5x^{-3} + \frac{7}{x} + \frac{2x^6}{3} - 9x + 26$ (ii) $y = 2x\ln\left(\frac{2}{x}\right)$

(iii) $y = \frac{4e^{-2x}}{\sin 5x}$ (iv) $y = \sqrt{1+5x^2-4x^3}$

Solutions

1. (i) $-15x^{-4} - 7x^{-2} + 4x^5 - 9$ (ii) $2\left[\ln\left(\frac{2}{x}\right) - 1\right]$

(iii) $\frac{-4e^{-2x}\left[2\sin(5x) + 5\cos(5x)\right]}{\sin^2(5x)}$ (iv) $\frac{x[5-6x]}{\sqrt{1+5x^2-4x^3}}$

2. If $y = a\cosh(x/a) + c$, where a and c are constants, show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{a}\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}$$

Solution

2. Use a hyperbolic identity $\cosh^2(t) - \sinh^2(t) = 1$.

3. A curve is given, parametrically, by

$$x = t - \sin\left(\frac{®t}{2}\right), \quad y = \cos\left(\frac{®t}{2}\right) - t^2$$

Find the x- and y – coordinates of the point P which corresponds to the parameter $t = 1$ and find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P.

Solution

3. $P = (0, \ -1)$, $-\frac{®}{2} - 2$

Chapter 8

4. Find the integrals:

(i) $\int_{-1}^{2}\left(4x^3 - x + 1\right)dx$ (ii) $\int_{0}^{®/4}\cos\left(2t - \frac{®}{4}\right)dt$ (iii) $\int\frac{x}{x^2+1}dx$

(iv) $\int ¯\sin(¯)\,d¯$ (v) $\int\frac{3}{(x+1)(x-2)}dx$ (vi) $\int_{0}^{1/2}\frac{1}{\sqrt{1-x^2}}dx$

Solutions

4. (i) $16\frac{1}{2}$ (ii) $\frac{1}{\sqrt{2}}$ (iii) $\frac{1}{2}\ln\left(x^2+1\right) + C$

(iv) $-¯\cos(¯) + \sin(¯) + C$ (v) $\ln\left|\frac{x-2}{x+1}\right| + C$ (vi) $\frac{®}{6}$

5. Evaluate each of the following definite integrals.

(a) $\int_0^{\frac{®}{3}} 8\cos^3(x)\sin(x)\,dx$ (b) $\int_1^4 \frac{\ln(x)}{\sqrt{x}}\,dx$

Solution

5. (a) $\frac{15}{8}$ (b) $4[\ln(4)-1]$

Chapter 7

6. Determine the dimensions of the rectangle of largest area with a fixed perimeter of 20 feet.

7. You are to construct an open (no top) rectangular box with a square base. Material for the base costs 30 cents/in.^{2} and material for the sides costs 10 cents/in.^{2}. If the box's volume is required to be 12 in.^{3}, what dimensions will result in the least expensive box?

Solutions

6. A square of edge 5 feet.
7. 2 in. by 3in.

Chapter 9

8. Find the area of the region bounded by the curve $y = 3x - x^2$ and the *x* axis.
9. *R* is the region bounded by the curves $y = 3 - \sin(x)$, $y = \sin(x)$, $x = 0$ and $x = ®$. Find the area of *R*.

10. Using Simpson's rule with 4 strips, find an approximate value of the integral $\int_0^1 e^{x^2}\,dx$.

11. Write down the Trapezoidal Rule approximation T_3 for $\int_1^4 x\cos(® / x)\,dx$. Leave your answer expressed as a sum involving cosines.
[T_3 in this question means consider 3 intervals].

12. Approximate $\int_0^2 \frac{x}{1+x^2}\,dx$ using the Trapezoidal Rule with $n = 4$ subintervals.

13. Find the area under the graph of $y = \ln x$ between $x = 1$ and $x = 3$.

Solutions

8. 9/2
9. $3® - 4$
10. 1.464 (3dp)
11. $\frac{1}{2}\left[\cos(®) + 2\left(2\cos(® / 2) + 3\cos(® / 3)\right) + 4\cos(® / 4)\right]$
12. 0.78077 (5dp)
13. $\ln(27) - 2$

Chapter 10

14. Given that $z_1 = 4 + j2$, $z_2 = 3 - j$ find (i) $3z_1 + 2z_2$ (ii) $z_1 z_2$

15. Given $z_1 = 2 + j$, $z_2 = 3 - j4$, find (a) $z_1 z_2$ (b) Both values of $\sqrt{z_1}$

16. Find the **four** solutions to the equation

$$z^4 = -2 - j2\sqrt{3}$$

and display them on an Argand diagram.

17. Determine all complex cube roots of $8i$.

18. Determine all complex numbers z such that $z^5 = -32$.

<u>Solutions</u>

14. (i) $18 + j4$ (ii) $14 + j2$

15. (a) $10 - j5$ (b) $5^{1/4}\angle(13.28°)$ and $5^{1/4}\angle(193.28°)$

16. $z_1 = \sqrt{2}\angle(60°)$, $z_2 = \sqrt{2}\angle(150°)$, $z_3 = \sqrt{2}\angle(240°)$ and $z_4 = \sqrt{2}\angle(330°)$

17. $z_1 = 2\angle(30°)$, $z_2 = 2\angle(150°)$ and $z_3 = 2\angle(270°)$

18. $z_1 = 2\angle(36°)$, $z_2 = 2\angle(108°)$, $z_3 = 2\angle(180°)$, $z_4 = 2\angle(252°)$ and $z_5 = 2\angle(324°)$

Chapter 11

19. Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$. Determine (i) $(\mathbf{A}-\mathbf{B})(\mathbf{A}+\mathbf{B})$ (ii) $\mathbf{A}^2 - \mathbf{B}^2$

20. Given the matrix

$$\mathbf{A} = \frac{1}{7}\begin{pmatrix} 3 & -2 & -6 \\ -2 & 6 & -3 \\ -6 & -3 & -2 \end{pmatrix}$$

(a) Compute $\mathbf{A}^2$ and $\mathbf{A}^3$.

(b) Based on these results, determine the matrices $\mathbf{A}^{-1}$ and $\mathbf{A}^{2004}$.

21. Let **A**, **B** and **C** be matrices defined by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & -1 & 1 & 0 \\ -2 & 1 & 2 & 3 \end{pmatrix}$$

Which of the following are defined?

$$\mathbf{A}^T,\ \mathbf{AB},\ \mathbf{B}+\mathbf{C},\ \mathbf{A}-\mathbf{B},\ \mathbf{CB},\ \mathbf{BC}^T,\ \mathbf{A}^2$$

Compute those matrices which are defined.

22. Find the determinant of the matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$.

23. The mesh equations of a circuit are given by:

$$\begin{aligned} i_1 + 3i_2 + 2i_3 &= 13 \\ 4i_1 + 4i_2 - 3i_3 &= 3 \\ 5i_1 + i_2 + 2i_3 &= 13 \end{aligned}$$

Determine the values of i_1, i_2 and i_3.

Solutions

19. (i) $-16\begin{pmatrix} 4 & 5 \\ 4 & 5 \end{pmatrix}$ (ii) $-4\begin{pmatrix} 15 & 17 \\ 19 & 21 \end{pmatrix}$

20. (a) $\mathbf{A}^2 = \mathbf{I}$ and $\mathbf{A}^3 = \frac{1}{7}\begin{pmatrix} 3 & -2 & -6 \\ -2 & 6 & -3 \\ -6 & -3 & -2 \end{pmatrix}$ (b) $\mathbf{A}^{-1} = \mathbf{A}$ and $\mathbf{A}^{2004} = \mathbf{I}$

21. $\mathbf{A}^T = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ -1 & 1 & 3 \end{pmatrix}$, $\mathbf{CB} = \begin{pmatrix} 0 & -2 \\ -1 & 1 \end{pmatrix}$, $\mathbf{A}^2 = \begin{pmatrix} 0 & -2 & -4 \\ 3 & 3 & 3 \\ 6 & 8 & 10 \end{pmatrix}$. $\mathbf{AB}$, $\mathbf{B}+\mathbf{C}$, $\mathbf{A}-\mathbf{B}$ and $\mathbf{BC}^t$ is **not** valid.

22. $\det(\mathbf{A}) = 165$

23. $i_1 = 1$, $i_2 = 2$ and $i_3 = 3$