Exercise **15(b)**

1. Find the first 5 convergents of the following simple continued fractions. Also write the 5th convergent in decimal format correct to 2dp:
   (a) $[1;\ 1,\ 1,\ 1,\ 1]$ (b) $[2;\ 2,\ 2,\ 2,\ 2]$ (c) $[1;\ 2,\ 3,\ 4,\ 5]$
   (d) $[6;\ 7,\ 8,\ 9,\ 10]$

2. Find the first 9 convergents of the irrational number *e* given by:
   $$e = [2;\ 1,\ 2,\ 1,\ 1,\ 4,\ 1,\ 1,\ 6,\ 1,\ 1, \cdots]$$
   Write the 9th convergent in decimal format correct to 6 dp.

3. Find the first 9 convergents of the golden ration ¤ given by:
   $$¤ = \frac{1+\sqrt{5}}{2} = [1;\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ \cdots]$$
   Write the 9th convergent in decimal format correct to 6 dp.

4. Determine the simple continued fraction of the following irrational numbers:
   (a) $\sqrt{5}$ (b) $\sqrt{3}$ (c) $\sqrt{7}$

5. Determine the simple continued fraction of the following numbers and write down the first 5 convergents:
   (a) $\frac{1+\sqrt{3}}{2}$ (b) $\frac{2\sqrt{3}+3}{3}$ (c) $\frac{2+\sqrt{7}}{3}$

6. Determine the irrational numbers given by the following simple continued fraction:
   (a) $\sim = [\langle 1,\ 2\rangle]$ (b) $\sim = [\langle 2,\ 1\rangle]$ (c) $\sim = [2,\ \langle 5,\ 1\rangle]$

Brief Solutions

1. (a) 1, 2, 3/2, 5/3, 8/5 and 1.60 (b) 2, 5/2, 12/5, 29/12, 70/29 and 2.41
(c) 1, 3/2, 10/7, 43/30, 225/157 and 1.43
(d) 6, 43/7, 350/57, 3193/520, 32280/5257 and 6.14
2. 2, 3, 8/3, 11/4, 19/7, 87/32, 106/39, 193/71, 1264/465 and 2.718278 (6 dp)
3. 1, 2, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34 and 1.617647 (6 dp)
4. (a) $[2;\ \langle 4\rangle]$ (b) $[1;\langle 1,\ 2\rangle]$ (c) $[2;\langle 1,\ 1,\ 1,\ 4\rangle]$
5. (a) $[\langle 1,\ 2\rangle]$; $C_1 = 1$, $C_2 = \frac{3}{2}$, $C_3 = \frac{4}{3}$, $C_4 = \frac{11}{8}$, $C_5 = \frac{15}{11}$
   (b) $[2;\ \langle 6,\ 2\rangle]$; $C_1 = 2$, $C_2 = \frac{13}{6}$, $C_3 = \frac{28}{13}$, $C_4 = \frac{181}{84}$, $C_5 = \frac{390}{181}$
   (c) $[\langle 1,\ 1,\ 1,\ 4\rangle]$; $C_1 = 1$, $C_2 = 2$, $C_3 = \frac{3}{2}$, $C_4 = \frac{14}{9}$, $C_5 = \frac{17}{11}$
6. (a) $\frac{1}{2}\left(1+\sqrt{3}\right)$ (b) $1+\sqrt{3}$ (c) $\frac{15+3\sqrt{5}}{10}$