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$A\|=6 \Rightarrow$ unique soln.

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Soln to Ex 1:

$$\det\begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix} = (2\times 4) - (0\times 3) = 8$$

Soln to Ex 1

$$BA = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} P & Q & R \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} P' & Q' & R' \\ 0 & 4 & 0 \\ 0 & 6 & 12 \end{pmatrix}$$

Area of $\Delta PQR = 3$

$\Delta P'Q'R' = \frac{1}{2}(4\times 8) = 24$

Area has been increased by $\frac{24}{3} = 8 = \det(B)$.

$$\det\begin{pmatrix} 3 & 1 \\ 4 & 4 \end{pmatrix} = (3\times 4) - (1\times 1) = 11.$$

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{\det(A)} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{\det A} \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Hence $A^{-1} = B = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(a) $A^{-1} = \det(A) = \det\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} = 10 + 3 = 13$

$$A^{-1} = \frac{1}{13}\begin{pmatrix} 5 & -3 \\ 1 & 2 \end{pmatrix}$$

(b) $\det(B) = \det\begin{pmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{pmatrix} = 2 + 1 = 3$

$$B^{-1} = \frac{1}{3}\begin{pmatrix} \sqrt{2} & -1 \\ 1 & \sqrt{2} \end{pmatrix}$$

$\det(C) = 11^2 - 11^2 = 0$ No inverse

$$W(\cos(x), \sin(x)) = \det \begin{pmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{pmatrix}$$

$$= \cos^2(x) + \sin^2(x) = 1$$