$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} ; \quad A^{10}$$

Chara eqn is $\det(A - \lambda I) = 0$.

$$\lambda^{10} + 3\lambda^{9} + 6\lambda^{8} \cdots = 0.$$

$$p(\lambda) = c_n \lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0$$
$$= 0.$$

How many roots does $p(\lambda) \neq 0$ have?

$$(x-1)^{101} = 0.$$

$$(\lambda - 1)^3 (\lambda - 2) = 0$$

$$\underbrace{\lambda_1 = 1, \ \lambda_2 = 1, \ \lambda_3 = 1}_{\lambda_{1,2,3} = 1} \ \& \ \lambda_4 = 2$$

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

Soln: We have

$$\det(A - \lambda I) = \det \begin{pmatrix} 2-\lambda & 1 & 3 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix}$$

$$= (2-\lambda)(2-\lambda)(2-\lambda) = 0$$

$$\lambda_{1,2,3} = 2$$

Let $\underline{y} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ belong to $\lambda_{1,2,3} = 2$. We have

$$(A - 2I)\,\underline{y} = \underline{0}$$

$$\begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$y + 3z = 0$$

$$y = -3z$$

Let $z = s$ ~~where~~ $(s \neq 0)$. Then $y = -3z = -3s$.

Let $x = t$.

$$\underline{y} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ -3s \\ s \end{pmatrix} = s \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

both s & t cannot be zero.

$$E_2 = \left\{ s\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$