

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad A^{10}$$

$$p(\lambda) = \lambda^n + c_{n-1} \lambda^{n-1} + \dots + c_1 \lambda + c_0 = 0$$

$$p(A) = A^n + c_{n-1} A^{n-1} + \dots + c_1 A + c_0 I = \underline{0}$$

Given that $A = \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix}$ illustrate C-H.

Soln: $\det(A - \lambda I) = \det \begin{pmatrix} 3-\lambda & 2 \\ 3 & 4-\lambda \end{pmatrix}$

$$= (3-\lambda)(4-\lambda) - 6$$

$$= \underline{=} (\lambda-3)(\lambda-4) - 6$$

$$= \lambda^2 - 7\lambda + 6 = 0$$

$$A^2 - 7A + 6I = 0$$

$$\boxed{A^2 - 7A + 6I}$$

$$7A - 6I = 7 \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 21-6 & 14-0 \\ 21-0 & 28-6 \end{pmatrix} = \begin{pmatrix} 15 & 14 \\ 21 & 22 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 15 & 14 \\ 21 & 22 \end{pmatrix}.$$

$$A = \begin{pmatrix} 0.4 & 0.2 \\ 0.3 & 0.6 \end{pmatrix} \quad A^{100} = \begin{pmatrix} 29.6 & 13.5 \\ 20.4 & 9.2 \end{pmatrix}$$

Ex 12: $A = \begin{pmatrix} -2 & -4 \\ 1 & 3 \end{pmatrix}$ Find A^4 .

Soln: $p(\lambda) = \lambda^2 - \lambda - 2 = 0$.

$$p(A) = A^2 - A - 2I = 0$$

$$A^2 = A + 2I$$

$$A^4 = (A^2)^2$$

$$= (A + 2I)(A + 2I)$$

$$= A^2 + 2A + 2A + 4I^2$$

$$= \cancel{A} + 4A + 2I$$

$$= A^2 + 4A + 4I$$

$$= A + 2I + 4A + 4I$$

$$A^4 = 5A + 6I$$

$$= 5 \begin{pmatrix} -2 & -4 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} -10+6 & -20+0 \\ 5+0 & 15+6 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -20 \\ 5 & 21 \end{pmatrix} = A^4$$

$$A = \begin{pmatrix} 10 & 15 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 6 \end{pmatrix} \cdot \text{Find } A^{-1}$$

$$P(\lambda) = \lambda^3 - 20\lambda^2 + 94\lambda - 60.$$

Soln:

$$A^3 - 20A^2 + 94A + 60I = \underline{0}.$$

$$AB = I$$

$$A^3 - 20A^2 + 94A = 60I$$

$$\frac{1}{60}(A^3 - 20A^2 + 94A) = I$$

$$A \left[\frac{1}{60}(A^2 - 20A + 94I) \right] = I$$

$\underbrace{\hspace{10em}}_{A^{-1} =}$