Page 497

$$\det(A-\lambda I) = \det\left[\begin{pmatrix} 1 & 0 & 4 \\ 0 & 4 & 0 \\ 3 & 5 & -3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right]$$

$$= \det\begin{pmatrix} 1-\lambda & 0 & 4 \\ 0 & 4-\lambda & 0 \\ 3 & 5 & -3-\lambda \end{pmatrix}$$

$$= (4-\lambda)\det\begin{pmatrix} 1-\lambda & 4 \\ 3 & -3-\lambda \end{pmatrix}$$

$$= (4-\lambda)\left[(1-\lambda)(-3-\lambda) - 12\right]$$

$$= (4-\lambda)\left[(-\cdot-)(\lambda-1)(\lambda+3) - 12\right]$$

$$= (4-\lambda)\left[\lambda^2 + 2\lambda - 3 - 12\right]$$

$$= (4-\lambda)\left[\lambda^2 + 2\lambda - 15\right]$$

$$= (4-\lambda)\left[(\lambda+5)(\lambda-3)\right] = 0$$

$\lambda_1 = 4$, $\lambda_2 = -5$ & $\lambda_3 = 3$

Let $\underline{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ belong to $\underline{\lambda_3 = 3}$.

$$(A - \lambda I)\underline{v} = \underline{0}$$

$$\left[\begin{pmatrix} 1 & 0 & 4 \\ 0 & 4 & 0 \\ 3 & 5 & -3 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}\right]\underline{v} = \underline{0}$$

$$\begin{pmatrix} -2 & 0 & 4 \\ 0 & 1 & 0 \\ 3 & 5 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$y = 0$$

$$-2x + 4z = 0 \Rightarrow 2x = 4z$$

$$x = 2z$$

$$3x - 6z = 0$$

Let $z = s$ where $s \neq 0$. Then $x = 2z = 2s$.

$$\underline{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2s \\ 0 \\ s \end{pmatrix} = s\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

If $\underline{u}$ is e. vector belonging to $\lambda = 3$.

$$A\underline{u} = \lambda \underline{u}$$

$$A(666\underline{u}) = 666(A\underline{u}) = 666(3\underline{u}) =$$

$$= 3(666\underline{u})$$

$$A\underline{v} = 3\underline{v}$$

<u>Proof</u>:

$$A\underline{u} = \lambda \underline{u}$$

$$A(k\underline{u}) = k(A\underline{u}) = k(\lambda \underline{u}) = \lambda(k\underline{u})$$

$k\underline{u}$ also belongs to λ.