Eigenvalues & Eigenvectors   Page 491

- determine e.value & e.vectors
- prove properties of — ——.

$$\boxed{A\underline{v} = \lambda \underline{v}}$$

$\underline{v} \neq 0$

λ is called eigenvalue
i-gun

$\underline{v}$ is called eigenvector.

$$A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$A\underline{v} = \lambda \underline{v}$$

$$\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \lambda_1 = 2.$$

$$\underline{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$A\underline{v} = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow \lambda_2 = 3$$

$$A\underline{y} = \lambda \underline{y}$$

$$A\underline{y} = \lambda I \underline{y}$$

$$A\underline{y} - \lambda I \underline{y} = \underline{0}$$

$$(A - \lambda I)\underline{y} = \underline{0}$$

$$\underline{y} \neq 0$$

∞ number of sol^n, $\Leftrightarrow$ $\boxed{\det(A - \lambda I) = 0.}$

Characteristic eqn.

λ, $\underline{y}$

Determine e. values & e. vectors of $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$.

<u>Sol^n</u>: We have

$$\det(A - \lambda I) = \det\left[\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right]$$

$$= \det \begin{pmatrix} 2-\lambda & 0 \\ 1 & 3-\lambda \end{pmatrix}$$

$$= (2-\lambda)(3-\lambda) - 0 = 0$$

$\lambda_1 = 2$ & $\lambda_2 = 3$.

Let $\underline{u} = \begin{pmatrix} x \\ y \end{pmatrix}$ be the e.vector belonging to $\underline{\underline{\lambda_1 = 2}}$.

We have

$$(A - \lambda I)\underline{u} = \underline{0}$$

$$\left[\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\right] \underline{u} = \underline{0}$$

$$\begin{pmatrix} 0 & 0 \\ \boxed{1 \quad 1} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x + y = 0$$

$$x = -y$$

Let $y = s \in \mathbb{R}$ but $s \neq 0$.

$$x = -s$$

$$\underline{u} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -s \\ s \end{pmatrix} = s\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (s \neq 0)$$

Let $\underline{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ belongs to $\lambda_2 = 3$.

$$(A - \lambda I)\underline{v} = \underline{0}$$

$$\left[\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\right] \underline{v} = \underline{0}$$

$$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$\left.\begin{aligned} -x &= 0 \\ x &= 0 \end{aligned}\right\} \Rightarrow x = 0 \text{ also } y = s$$

where $s \neq 0$.

$$\underline{v} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ s \end{pmatrix} = s \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A\underline{v} = 3\underline{v}$$

$$A\underline{y} = 2\underline{y}.$$