Product Rule GD. Page 296.

$$\frac{d}{dx}\left[x^2 \ln(x)\right] \neq \frac{d}{dx}(x^2) \times \frac{d}{dx}(\ln(x))$$

$$\boxed{\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}}$$

$y = x\cos(x)$ find $\frac{dy}{dx}$.

Soln: Let $u = x$ $\quad v = \cos(x)$

$$\frac{du}{dx} = 1, \qquad \frac{dv}{dx} = -\sin(x)$$

$$\frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

$$= [1 \times \cos(x)] + x[-\sin(x)]$$

$$= \cos(x) - x\sin(x)$$

Let $y = x^2\ln(x)$ find $\frac{dy}{dx}$.

Soln, Let $u = x^2$ $\quad v = \ln(x)$

$$\frac{du}{dx} = 2x, \qquad \frac{dv}{dx} = \frac{1}{x}.$$

$$\frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

$$= 2x\ln(x) + x^{\cancel{2}}\left(\frac{1}{\cancel{x}}\right)$$

$$= 2x\ln(x) + x$$

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$$E = L\frac{di}{dt}$$

$L = 5\times10^{-3}$ $\quad i = te^{-t}$. Find E.

Soln:

$$\frac{d}{dt}\left(te^{-t}\right) =$$

Let $u = t$, $\quad v = e^{-t}$

$$\frac{du}{dt} = 1, \quad \frac{dv}{dt} = -e^{-t}$$

$$\frac{di}{dt} = \frac{du}{dt}v + \frac{dv}{dt}u$$

$$= e^{-t} + -te^{-t}$$

$$= e^{-t}[1-t]$$

$$E = (5\times10^{-3})\,e^{-t}[1-t]$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{\sin(x)}{x^2}\right] = ?$$

$$\parallel$$

$$x^{-2}\sin(x)$$

$$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}}$$

Soln: Let $u = \sin(x)$ $\quad v = x^2$

$$\frac{du}{dx} = \cos(x) \qquad \frac{dv}{dx} = 2x$$

$$\frac{d}{dx}\left[\frac{\sin(x)}{x^2}\right] = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$$

$$= \frac{x^2\cos(x) - 2x\sin(x)}{(x^2)^2}$$

$$= \frac{\cancel{x}\left[x\cos(x) - 2\sin(x)\right]}{x^{\cancel{4}3}}$$

$$= \frac{x\cos(x) - 2\sin(x)}{x^3}$$

Show $\frac{d}{dx}\left[\tan(x)\right] = \sec^2(x)$

$$\frac{d}{dx}\left[\tan(x)\right] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$$

Let $u = \sin(x)$ $\quad v = \cos(x)$

$\frac{du}{dx} = \cos(x)$ $\quad \frac{dv}{dx} = -\sin(x)$

$$\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{\cos(x).\cos(x) - \sin(x)(-\sin}{\left[\cos(x)\right]^2}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$ because $\frac{1}{\cos}$