

Solve

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = f(x) \text{ where } f(x) \neq 0.$$

$$y = f(x) + g(x)$$

$$y = y_c + \underbrace{y_p}_{\text{particular } \int} = x^2$$

$$y = ax^2 + bx + c$$

$$f(x) = e^{kx}$$

$$y = C e^{kx}$$

$$\frac{d^2 i}{dt^2} + (12 \times 10^6) \frac{di}{dt} + (36 \times 10^{12}) i = 5 \times 10^9$$

Soln:

$$m^2 + (12 \times 10^6) m + (36 \times 10^{12}) = 0$$

$$m = -(6 \times 10^6) \text{ Real \& Equal}$$

$$i = (A + Bt) e^{-(6 \times 10^6)t}$$

$$i = \underbrace{I = C}_{(A + Bt) e^{-(6 \times 10^6)t}} + C$$

$$I' = 0$$

$$I'' = 0$$

$$(36 \times 10^{12}) C = 5 \times 10^9 \quad (2)$$

$$C = \frac{5 \times 10^9}{36 \times 10^{12}} = \underline{\underline{\frac{5}{36} \times 10^{-3}}}$$

$$i = \textcircled{i_0} + I$$

$$= (A + Bt) e^{-(6 \times 10^8)t} + \left(\frac{5}{36} \times 10^{-3} \right)$$

Solve

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \textcircled{3e^{2x}}$$

Soln:

$$y = y_c + y$$

$$y_c = \textcircled{Ae^{2x}} + Be^{-x}$$

$$y = \textcircled{Ce^{2x}}$$

$$y' = 2Ce^{2x}$$

$$y'' = 4Ce^{2x}$$

$$4Ce^{2x} - 2Ce^{2x} - 2Ce^{2x} = 3e^{2x}$$

$$0 = 3e^{2x}$$

$$\underline{y = Cx e^{2x}}$$

(3)

$$C = 1$$

$$y = y_c + Y$$

$$= A e^{2x} + B e^{-x} + x e^{2x} .$$