Differential Eqns of the form

(1)

$$\boxed{\frac{d^2y}{dx^2} + k^2y = 0}$$

Soln,

$$m^2 + k^2 = 0$$

$$m^2 = -k^2$$

$$m = \sqrt{-k^2} = \pm jk$$

$$= 0 \pm jk$$

$$m = \alpha \pm j\beta \qquad \alpha = 0, \ \beta = k$$

$$y = e^{\alpha x}\left[A\cos(\beta x) + B\sin(\beta x)\right]$$

$$\alpha = 1$$

$$\boxed{y = A\cos(\beta x) + B\sin(\beta x)}$$

$$y = A\cos(kx) + B\sin(kx)$$

$$\frac{d^2y}{dx^2} + k^2y = 0$$

$$y = A\cos(kx) + B\sin(kx).$$

$$\frac{d^2x}{dt^2} + 25x = 0$$

when $t=0$, $x=1$ & $\frac{dx}{dt} = 10$.

Soln:

$$\frac{d^2x}{dt^2} + 5^2 x = 0$$

$$x = A\cos(5t) + B\sin(5t)$$

$$1 = A\underbrace{\cos(0)}_{=1} + B\underbrace{\sin(0)}_{=0}$$

Diff ↓

$$1 = A$$

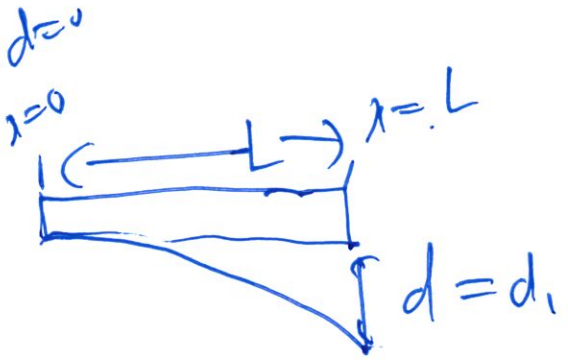
$$\frac{dx}{dt} = -5A\sin(5t) + 5B\cos(5t)$$

$$10 = -5A\underbrace{\sin(0)}_{=0} + 5B\underbrace{\cos(0)}_{=1}$$

$$10 = -5A\,0 + 5B \Rightarrow B = \frac{10}{5} = 2$$

$$x = \cos(5t) + 2\sin(5t)$$

IVP.



$$\frac{d^2y}{dx^2} - k^2 y = 0.$$

Soln:

$$m^2 - k^2 = 0$$

$$m^2 = k^2$$

$$m = \pm k \qquad m_1 = k, \quad m_2 = -k$$

↑ real & different.

$$y = Ae^{kx} + Be^{-kx}$$
