Injective & Surjective Functions

\[ f(x) = x^2 \]

Many \quad \rightarrow \quad \text{one}

Let \( f: \mathbb{R} \rightarrow \mathbb{R} \) be given by \( f(x) = 2x \).

Find the image of

\(-3, -2, -1, 0, 1, 2, 3\).

**Solution:**

One \quad \rightarrow \quad \text{one}.

One \rightarrow \text{one} is called an injective function.
\[ f(x) = \frac{2x}{x-3} \]
\[ f(y) = \frac{2y}{y-3} = f(x) \]
\[ 2x(y-3) = 2y(x-3) \]
\[ 2xy - 6x = 2xy - 6y \]
\[ -6x = -6y \]

\[ x = y \]

Hence \( f \) is an injective function.

Let \( f: \mathbb{R}^+ \to \mathbb{R} \) be given by

\[ f(x) = x^2 + 2 \]

Test \( f \) for injection.

So \( y \):

\[ f(x) = x^2 + 2 = y^2 + 2 = f(y) \]

\[ x^2 = y^2 \]

\[ x = \pm y \]

\[ x = y \text{ or } x = -y \]

\( x = y \) or \( x = -y \) \( \times \)

Hence \( x = y \) so injective.
Let \( f : \mathbb{R} \to \mathbb{R} \) be given by 
\[
f(x) = x^2.
\]

Test \( f \) for injectivity.

Solv:
\[
f(x_1) = x_1^2 = y^2 = f(y) \]
\[
x = \pm y
\]
\[
x = y \quad \text{or} \quad x = -y.
\]

Hence \( f \) is not injective.

\[
f(-6) = 36 = f(6).
\]

Let \( f : \mathbb{R} \to \mathbb{R} \) be 
\[
f(x) = x + 1.
\]

Test \( f \) for surjectivity.

Solv:
\[
y = f(x) = x + 1.
\]
\[
x = y - 1 \quad \text{or} \quad x \in \mathbb{R}.
\]

Hence \( f \) is surjective.