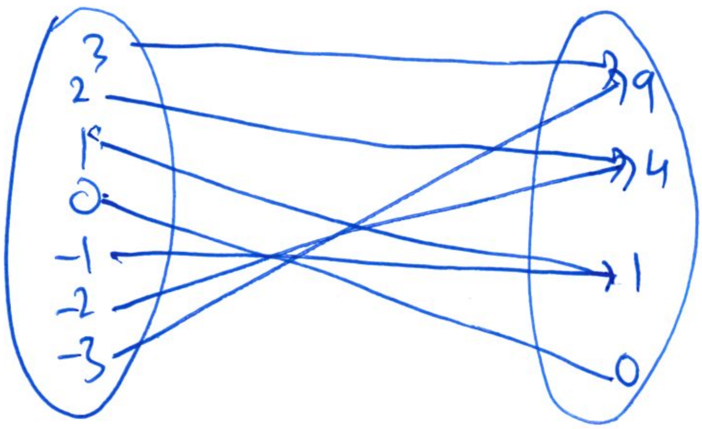
Injective & Surjective Functions

$f(x) = x^2$



Many $\longrightarrow$ one.

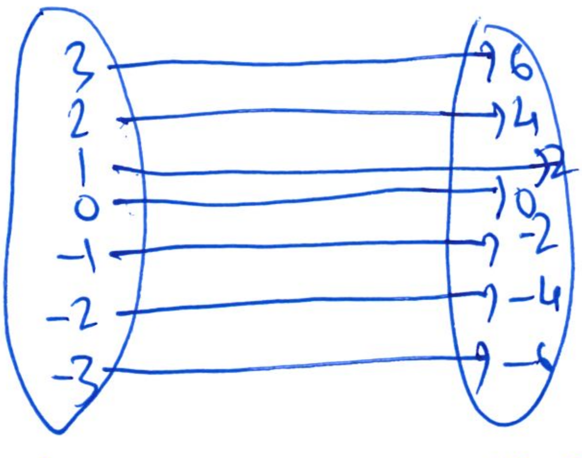
↑ unique.

$f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 2x$.

Find the images of

$-3, -2, -1, 0, 1, 2, 3.$

Soln,



One $\longrightarrow$ one.

One $\rightarrow$ one is called an injective function.

$$f(x) = \frac{2x}{x-3}$$

$$f(x) = \frac{2x}{x-3} = \frac{2y}{y-3} = f(y)$$

$$2x(y-3) = 2y(x-3)$$

$$2xy - 6x = 2xy - 6y$$

$$-6x = -6y$$

$$\downarrow$$

$$x = y$$

Hence f is an injective function.

Let $f: \mathbb{R}^+ \to \mathbb{R}$ be given by

$$f(x) = x^2 + 2.$$

Test f for injection.

Sol^n:

$$f(x) = x^2 + 2 = y^2 + 2 = f(y)$$

$$x^2 = y^2$$

$$x = \sqrt{y^2} = \pm y$$

$$x = y \text{ or } x = -y \times$$

Hence $x = y$ so injective.

Let $f: \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = x^2.$$

Test f for injection.

Sol^n:

$$f(x) = x^2 = y^2 = f(y)$$

$$x = \pm y$$

$$x = y \text{ or } x = -y.$$

Hence f is not injective.

$$f(-6) = 36 = f(6).$$

Let $f: \mathbb{R} \to \mathbb{R}$ be

by $f(x) = x+1.$

Test f for surjection.

Sol^n: Let $y = f(x) = x+1.$

$$x = y - 1 \quad . \; x \in \mathbb{R}.$$

Hence f is surjective.