Exercise **4(e)**

In this exercise lower case letters such as x, y, z *etc are real numbers.*

1. Determine the following: $|e|$, $|-e|$, $\left|-\sqrt{2}\right|$, $|-6-7|$ and $\left|\cos\left(\frac{3®}{4}\right)\right|$.
2. Solve the following inequalities:

a. $|x|<1$

b. $|x|<®$

c. $|x-1|<1$

d. $|x-5|\le 2$

e. $|x+5|<2$

f. $|x+1|\le 1$

3. Solve the following inequalities:

(a) $|2x-1|<2$

(b) $|3x-3|<®$

(c) $|x-a|<¢$

(d) $|x-2a|<¢$

4. Prove that if $n\in\mathbb{N}$ then $\left|\frac{1}{n}\right|=\frac{1}{n}$.
5. Prove that $|x|=\sqrt{x^2}$.
6. Prove that $|x-y|=|y-x|$.
7. Prove that $|x-y|\le|x|+|y|$.
8. Prove that $\left|\frac{1}{x}\right|=\frac{1}{|x|}$ where $x\neq 0$.
9. Prove that $\left|\frac{x}{y}\right|=\frac{|x|}{|y|}$ where $y\neq 0$.
10. Give another proof of the triangle inequality, $|x+y|\le|x|+|y|$.

11. Prove that $|x_1 + x_2 + x_3 + ... + x_n| \le |x_1| + |x_2| + |x_3| + ... + |x_n|$.

12. Prove other triangle inequalities such as

(a) $|x - y| \ge |x| - |y|$ (b) $\big||x| - |y|\big| \le |x - y|$

13. Prove that if for every $£ > 0$ we have $|x - y| < \frac{£}{2}$ and $|y - z| < \frac{£}{2}$ then $x = z$.

14. Prove that if for every $£ > 0$ we have $|x - y| < \frac{£}{3}$, $|y - z| < \frac{£}{3}$ and $|z - w| < \frac{\varepsilon}{3}$ then $x = w$.

Solutions 4(e)

1. $|e| = e$, $|-e| = e$, $\left|-\sqrt{2}\right| = \sqrt{2}$, $|-6-7| = 13$ and $\left|\cos\left(\frac{3®}{4}\right)\right| = \frac{1}{\sqrt{2}}$.

2. a. $-1 < x < 1$ b. $-® < x < ®$ c. $0 < x < 2$

d. $3 < x < 7$ e. $-7 < x < -3$ f. $-2 < x < 0$

3. (a) $-0.5 < x < 1.5$ (b) $1 - \frac{®}{3} < x < 1 + \frac{®}{3}$

(c) $a - ¢ < x < a + ¢$ (d) $2a - ¢ < x < 2a + ¢$

4. Since $\frac{1}{n} > 0$ therefore result follows.

5. Consider $x \ge 0$ and $x < 0$ separately.

6. Rewrite $x - y = -(y - x)$ and then take the modulus.

7. Rewrite $x - y = x + (-y)$ and then use the triangle inequality.

8. Consider $x \ge 0$ and $x < 0$ separately.

9. Rewrite $\frac{x}{y} = x\left(\frac{1}{y}\right)$ and then use the result of question 8.

10. Use propositions (4.15) and (4.16).

11. Use mathematical induction.

12. Use the triangle inequality (4.19) or square each side and take the square root.

13. Write $|x - z| = |x - y + y - z|$ and then use the triangle inequality.

14. Write $|x - w| = |x - y + y - z + z - w|$ and then use the triangle inequality.