Tough Nut to Crack – Integration Solution V:

Problem

Show that

$$\int \frac{dx}{\sqrt{x}-1} = 2\left[\sqrt{x} + \ln\left|\sqrt{x}-1\right|\right] + C$$

There is **no reward** for differentiating the Right Hand Side and showing that it is equal to the integrand on the Left Hand Side.

Solution

Let $u = \sqrt{x}$ then $u^2 = x$ and we have

$$x = u^2 \quad \Rightarrow \quad \frac{\mathrm{d}x}{\mathrm{d}u} = 2u \quad \Rightarrow \quad dx = 2u\,\mathrm{d}u$$

Substituting these into the given integrand results in

$$\int \frac{dx}{\sqrt{x}-1} = \int \left(\frac{2u}{u-1}\right)\mathrm{d}u$$

$$= 2\int \left(1 + \frac{1}{u-1}\right)\mathrm{d}u \qquad \left[\text{By using partial fractions}\right]$$

$$= 2\left[u + \ln\left|u-1\right|\right] + C$$

$$= 2\left[\sqrt{x} + \ln\left|\sqrt{x}-1\right|\right] + C \qquad \left[\text{Substituting } u = \sqrt{x}\right]$$

This is our required result.

These solutions were provided by Andrew Murphy, E Brodie, Jeremy Abraham and Alia Razaq.