Engineering Maths,
Bonus Question, Matrices.

December 8, 2012

One of the research interests of physicists at the university is the optical scattering properties of ice crystals. In a nutshell, this means looking at the way ice crystals have on the motion of light. This is an important quantity to measure as clouds contain ice crystals which scatter the infra red radiation from the sun. in Fig 1 the triangles represent ice crystals, we see that they are randomly

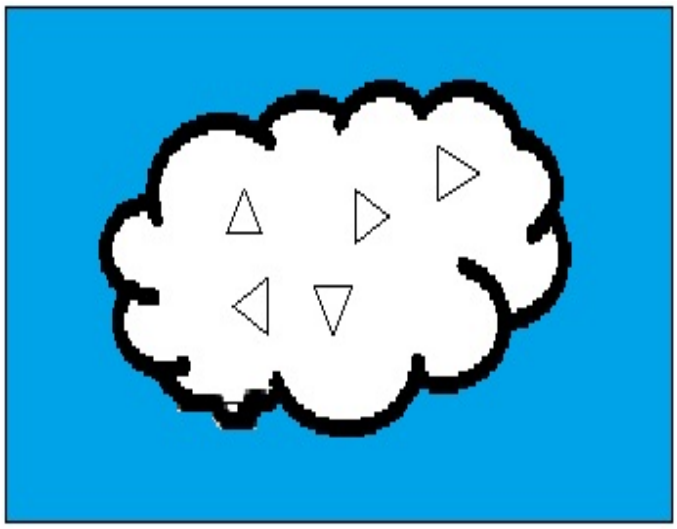

Figure 1: A cloud containing ice crstals

orientated. In order to model this random orientation we use matrices, more specifically, a special type of matrix called a rotation matrix. We're going to investigate this matrix here.
Each vertex (corner) of one of our triangular ice crystals has an (x, y) coordinate. A triangle could have coordinates $(0, 0), (2, 0), (1, 1)$. We write these as one matrix as seen below:

$$\mathbf{V} = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Hopefully, you can see that each column of the matrix $\mathbf{V}$ is an (x, y) coordinate of one the vertices. We now define a rotation matrix $\mathbf{R}$

$$\mathbf{R} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

We're going to consider the matrix $\mathbf{R}$ when $\theta = 45°$

1. Draw the triangle described by the matrix **V**
2. Perform the multiplication **RV**
3. The new matrix you've created are the new (x, y) coordinates, plot these points
4. describe what effect the matrix had on our ice crystal.

One other quantity of interest with matrices is a property called the determinant, for a 2×2 matrix with elements:

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The determinant (det$\mathbf{A}$) would be given by $ad - bc$. With that in mind, can you please find (det$\mathbf{R}$) (with θ being θ not $45°$). The answer you get is true for all rotation matrices.