

Mathematics, Bonus Question

December 3, 2012

This one is particularly tough, you should be ok though. You may find part 3 very hard given that you may not be familiar with graphical programs. However, you could use excel to get a rough idea and then just sketch...

However you will need the following:

$$j = \sqrt{-1}, \quad j^2 = -1$$
$$\int e^{jx} dx = \frac{e^{jx}}{j}$$

Note that j is the infamous imaginary unit, don't worry too much about its definition, we'll cover that in the coming weeks. For now just use it. The Fourier Transform is a very powerful tool used extensively in applied mathematics. Definitions can vary but we'll use this version:

$$\tilde{F}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-jkx} dx$$

We can replace the infinity signs by the limits of integration. Consider, the function below.

$$f(x) = \begin{cases} 1, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

using the definition of the Fourier Transform, can you please:

1. Show that $j^{-1} = -j$
2. Sketch the function $f(x)$ for $-5 < x < 5$
3. find the Fourier transform of the function $f(x)$, that is, evaluate $\tilde{F}(x)$
4. Plot the resulting function using the same limits as before, you will need to use a computer