Exercise 6(e)

1. Solve the following linear system of equations by using Cramer's rule:

(a) $\begin{aligned} x+2y+\ z&=2 \\ 3x-3y-2z&=0 \\ 4x-2y-5z&=-9 \end{aligned}$ (b) $\begin{aligned} 2x+2y\ -z&=7 \\ -3x+5y\ -z&=0 \\ 7x+9y+13z&=10 \end{aligned}$ (c) $\begin{aligned} x+5y-6z&=1 \\ x-3y+7z&=3 \\ 3x+4y-2z&=2 \end{aligned}$

2. Determine the solution of the following system by using Cramer's rule:

$$\begin{aligned} x-2y+3z+5w&=7 \\ 2x\ -y-3z+2w&=9 \\ 5x+3y+7z+11w&=3 \\ 2x-4y+6z+10w&=8 \end{aligned}$$

3. Find the solution of the following linear system by applying Cramer's rule:

(a) $\begin{aligned} 3x+2y+7z&=7 \\ 2x\ -y-3z&=-3 \\ 7x+4y+5z&=5 \end{aligned}$ (b) $\begin{aligned} 11x-3y+2z&=-3 \\ 7x\ -y+5z&=-1 \\ 3x+10y+8z&=10 \end{aligned}$ (c) $\begin{aligned} 5x+2y+11z&=5 \\ -3x-4y+6z&=-3 \\ -2x+5y+4z&=-2 \end{aligned}$

4. Consider a general linear system $\mathbf{Ax}=\mathbf{b}$ where **A** is a n by n matrix and **x** and **b** are n by 1 column vectors. If the *j*th column, where $j=1,\ 2,\ 3,\ 4,\ \cdots,\ n$, of matrix **A** consists of the column **b** and $\det(\mathbf{A})\neq 0$ then show that the solution of the system is

$$x_1=0,\ x_2=0,\cdots,\ x_{j-1}=0,\ x_j=1,\ x_{j+1}=0,\ \cdots \text{ and } x_n=0$$

5. By applying Cramer's rule, show that the linear system $\mathbf{Ax}=\mathbf{O}$ has the trivial solution.

Brief Solutions to Exercise 6(e)

1. (a) $x=1,\ y=-1$ and $z=3$ (b) $x=2,\ y=1$ and $z=-1$
(c) $x=-2,\ y=3$ and $z=2$

2. The determinant of matrix **A** is $\det(\mathbf{A})=0$ therefore *cannot* use Cramer's rule.

3. (a) $x=0,\ y=0$ and $z=1$ (b) $x=0,\ y=1$ and $z=0$
(c) $x=1,\ y=0$ and $z=0$

4. Use the proposition that the determinant of the matrix with two identical columns is zero.