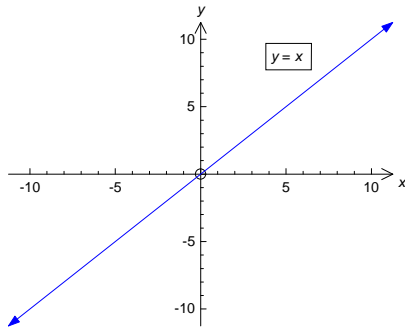


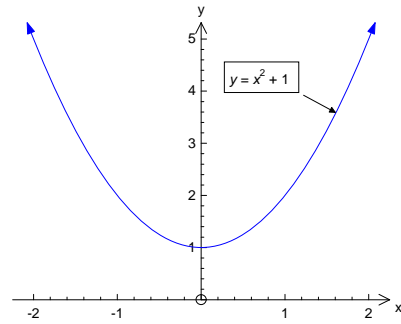
**Exercise 3(f)**

Questions 8, 10 and 11 are Workbook questions.

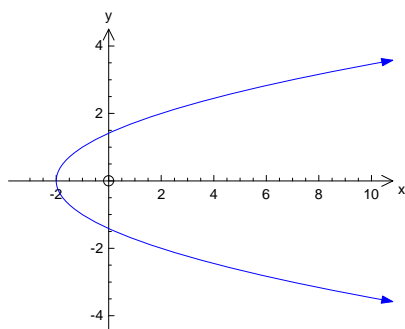
1. Determine which of the following graphs are functions:



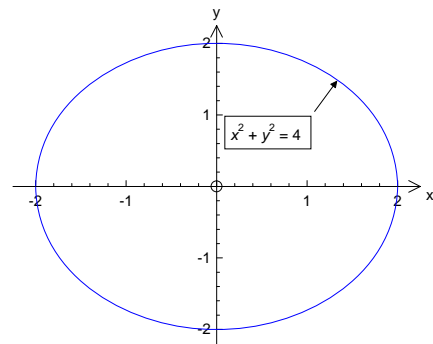
(a)



(b)

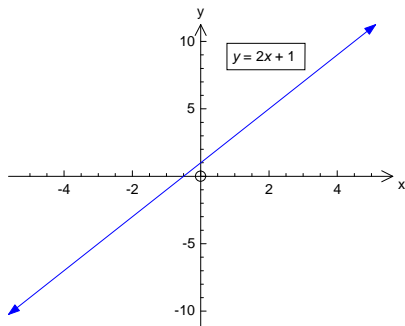


(c)

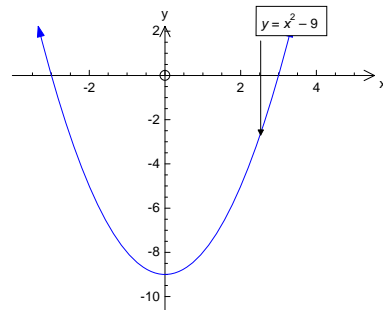


(d)

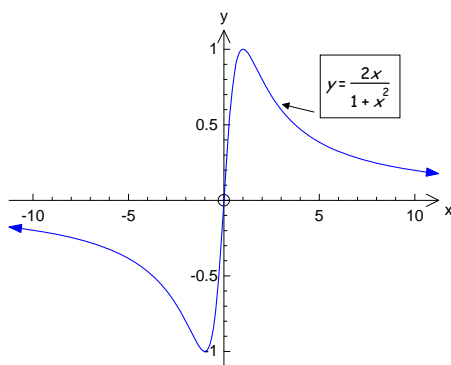
2. Decide which of the following functions are injective:



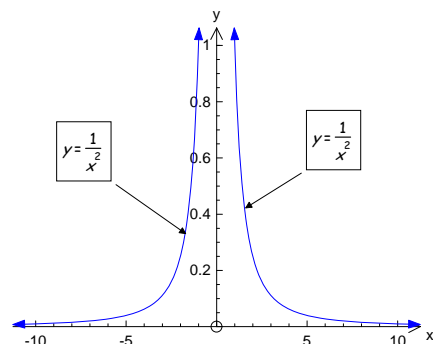
(a)



(b)

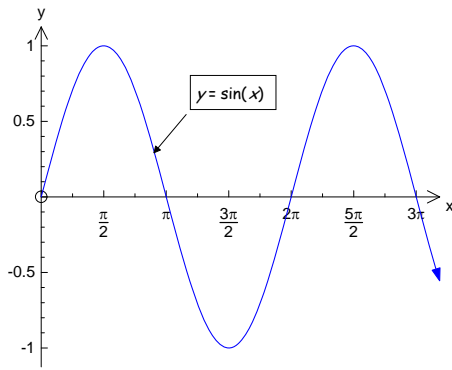


(c)

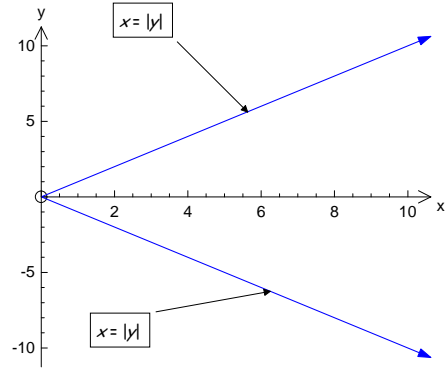


(d)

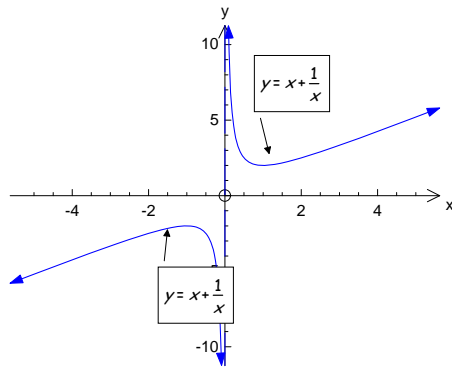
3. Decide which of the following functions are injective:



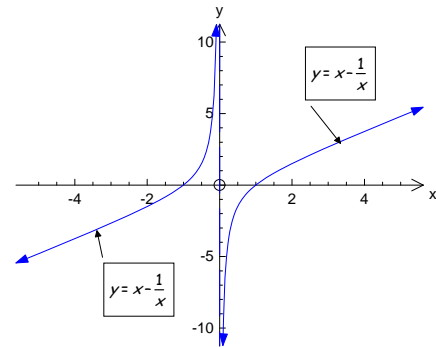
(a)



(b)

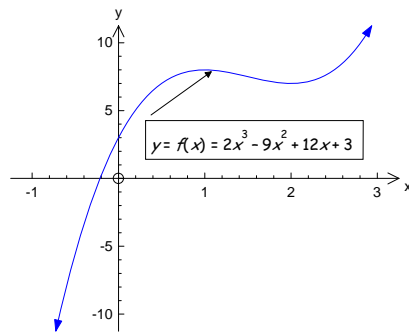


(c)

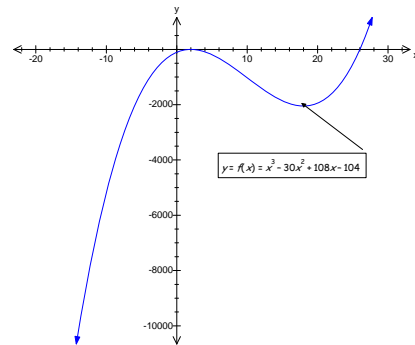


(d)

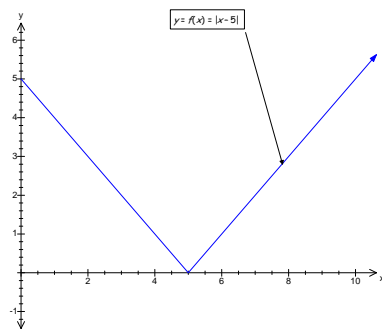
4. Test the following functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  for injection and surjection:



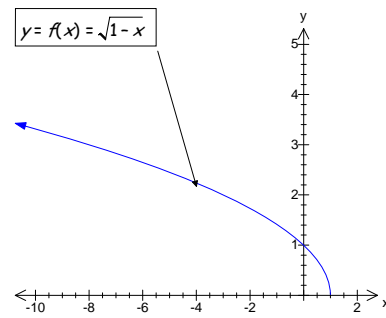
(a)



(b)



(c)



(d)

5. Test the following functions for injection and surjection by the

(i) graphical method      (ii) algebraic method

- (a)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x$   
 (b)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x + 1$   
 (c)  $f : \mathbb{R}^+ \rightarrow \mathbb{R}$  given by  $f(x) = \sqrt{x}$   
 (d)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x^2 + 1$   
 (e)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x^3 + 1$   
 (f)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x^3 - 1$   
 (g)  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = \sin(x)$

6. Test the following functions for injection and surjection by the

(i) graphical method      (ii) algebraic method

- (a)  $f : \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(n) = n$   
 (b)  $f : \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(n) = 2n + 1$   
 (c)  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $f(x) = 2x + 1$   
 (d)  $f : \mathbb{Z} \rightarrow \mathbb{Z}$  defined by  $f(x) = x + 1$

7. Let  $A = \{x \mid x \in \mathbb{R} \text{ and } -10 \leq x \leq 10\}$  and  $B = \{x \mid x \in \mathbb{R} \text{ and } -26 \leq x \leq 34\}$ .

Let  $f : A \rightarrow B$  be defined by the formula

$$f(x) = 3x + 4$$

Sketch the graph of  $f$  and use this to determine whether the function  $f$  is injective and/or surjective. Confirm your results algebraically.

**8. Let  $A = \{x \mid x \in \mathbb{R} \text{ and } -3 \leq x \leq 5\}$  and  $B = \{x \mid x \in \mathbb{R} \text{ and } 2 \leq x \leq 27\}$ .**

**Let  $f : A \rightarrow B$  be defined by the formula**

$$f(x) = x^2 + 2$$

**Sketch the graph of  $f$  and use this to determine whether the function  $f$  is injective and/or surjective. Confirm your results algebraically.**

9. Let  $f : \mathbb{R} \rightarrow \{0, 1\}$  be a function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Sketch this graph and test  $f$  for injection and/or surjection. Verify your results algebraically.

**10. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by the formula**

$$f(x) = (x - 1)^2$$

**Sketch the graph of  $f$  and use this to determine whether the function  $f$  is injective and/or surjective. Confirm your results algebraically.**

11. Let  $A = \{1, 2, 3\}$  and  $B = \{-1, 2, 7\}$ .

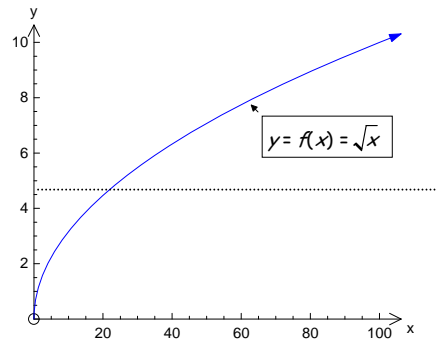
Let  $f : A \rightarrow B$  be defined by the formula

$$f(x) = x^2 - 2$$

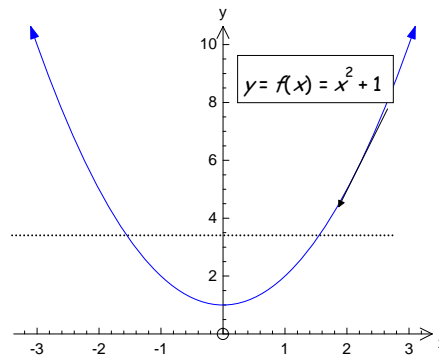
Sketch the graph of  $f$  and use this to determine whether the function  $f$  is injective and/or surjective.

### Solutions to Exercise 3f

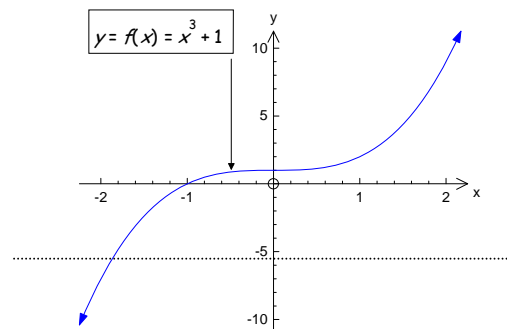
1. (c) and (d) are not functions.
2. Only (a) is injective.
3. Only (b) is injective.
4. (a) Surjective but not injective.  
(b) Surjective but not injective.  
(c) Neither injective nor surjective.  
(d) Injective but not surjective.
5. (a) Both injective and surjective.  
(b) Both injective and surjective.  
(c) Injective but not surjective.



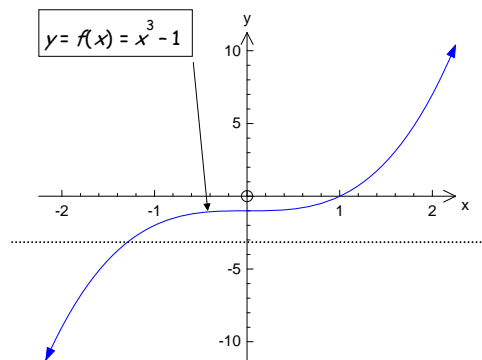
(d) Neither injective nor surjective.



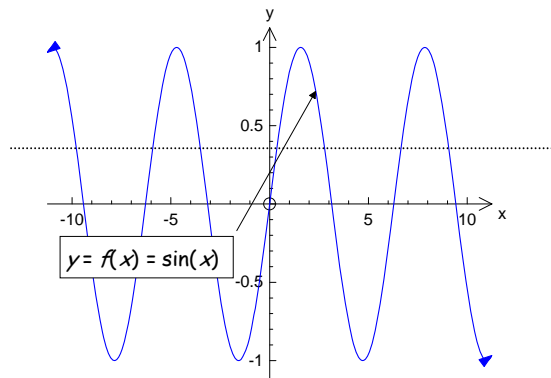
(e) Both Injective and Surjective.



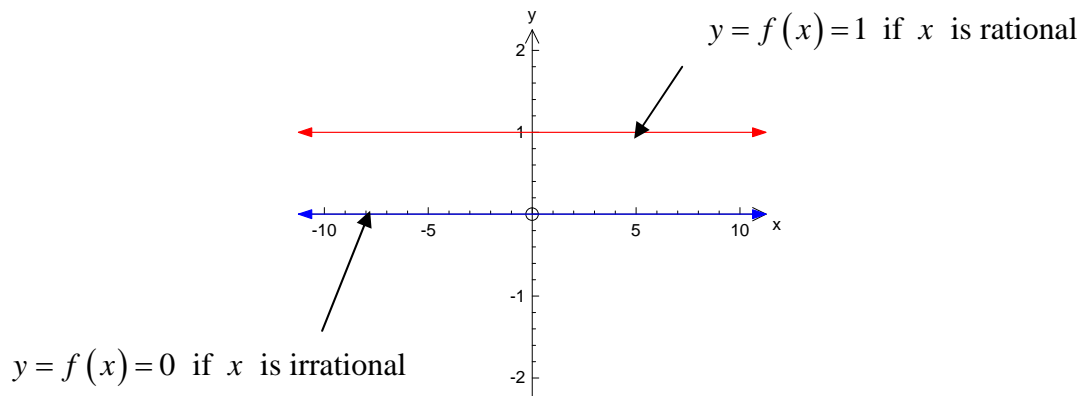
(f) Both Injective and Surjective.



(g) Neither injective nor surjective.



6. (a) Both injective and surjective.
  - (b) Injective not surjective. ( $f(n) = 2n + 1$  is an odd number).
  - (c) Injective not surjective. ( $f(x) = 2x + 1$  is an odd number).
  - (d) Both injective and surjective.
7. Both injective and surjective.
  8. Workbook Question.
  9. Surjective only.



10. Workbook Question.
11. Workbook Question.