

Exercise 2a

1. Write the following series in \sum notation:

a) $1 + \sqrt{2} + \sqrt{3} + 2 + \dots$

b) $2 + 4 + 6 + 8 + \dots$

c) $1 + 3 + 5 + 7 + \dots$

d) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$

e) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$

f) $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \dots$

2. Show that each of the following series convergent and determine its sum:

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$ c) $\sum_{n=1}^{\infty} \left(\frac{1}{\pi^n}\right)$

d) $\sum_{n=1}^{\infty} \left(\frac{1}{m}\right)^n$ where $m > 1$

3. Determine whether the following series is convergent. If it is convergent then find its sum.

a) $\sum_{n=1}^{\infty} \left(\frac{1}{2^{2n-1}}\right)$ b) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$

c) $\sum_{n=1}^{\infty} (e)^n$ d) $\sum_{n=1}^{\infty} 10 \left(\frac{1}{3}\right)^n$

4. Determine which of the following series converges. If it does converge then find its sum.

a) $8 + 4 + 2 + 1 + \dots$

b) $3 + 6 + 12 + 24 + \dots$

c) $16 + 12 + 9 + \frac{27}{4} + \dots$

5. Show that the following series converges and find the sum in each case:

a) $\sum_{n=1}^{\infty} \frac{1}{x^n}$ where $|x| > 1$

b) $\sum_{n=1}^{\infty} \left(\frac{x^n}{2^n}\right)$ where $|x| < 2$

c) $\sum_{n=1}^{\infty} \frac{1}{(1+x)^n}$ where $x > 0$

d) $\sum_{n=1}^{\infty} \frac{1}{(1+x^2)^n}$ where $x \neq 0$

Solutions 2a

1. a) $\sum_{n=1}^{\infty} \sqrt{n}$ b) $\sum_{n=1}^{\infty} (2n)$ c) $\sum_{n=1}^{\infty} (2n-1)$

d) $\sum_{n=1}^{\infty} \left[\frac{(-1)^{n+1}}{n} \right]$ e) $\sum_{n=0}^{\infty} \left(\frac{1}{3} \right)^n$ f) $\sum_{n=1}^{\infty} \left(\frac{2}{3} \right)^n$

2. a) $\sum_{n=0}^{\infty} \left(\frac{1}{3} \right)^n = \frac{1}{2}$ b) $\sum_{n=0}^{\infty} \left(\frac{1}{4} \right)^n = \frac{1}{3}$

c) $\sum_{n=0}^{\infty} \left(\frac{1}{\pi^n} \right) = \frac{1}{\pi-1}$ d) $\sum_{n=0}^{\infty} \left(\frac{1}{m} \right)^n = \frac{1}{m-1}$

3. a) Converges, $\sum_{n=1}^{\infty} \left(\frac{1}{2^{2n-1}} \right) = \frac{2}{3}$

b) Diverges

c) Diverges

d) Converges, $\sum_{n=1}^{\infty} 10 \left(\frac{1}{3} \right)^n = 5$

4. a) Converges $\sum_{n=0}^{\infty} 8 \left(\frac{1}{2} \right)^n = 16$

b) Diverges, $\sum_{n=0}^{\infty} 3(2)^n$

c) Converges, $\sum_{n=0}^{\infty} 16 \left(\frac{3}{4} \right)^n = 64$

5. a) $\sum_{n=1}^{\infty} \frac{1}{x^n} = \frac{1}{x-1}$

b) $\sum_{n=1}^{\infty} \left(\frac{x}{2} \right)^n = \frac{x}{2-x}$

c) $\sum_{n=1}^{\infty} \frac{1}{(1+x)^n} = \frac{1}{x}$

d) $\sum_{n=1}^{\infty} \frac{1}{(1+x^2)^n} = \frac{1}{x^2}$