Exercise 1(g)

1. Show that for all natural numbers, \( n \)
\[
2 + 4 + 6 + ... + 2n = n(n + 1)
\]

2. Prove that for all natural numbers, \( n \)
\[
2 + 5 + 8 + ... + (3n - 1) = \frac{1}{2} n(3n + 1)
\]

3. Prove that for all natural numbers, \( n \)
\[
1^3 + 2^3 + 3^3 + ... + n^3 = \frac{1}{4} n^2 (n + 1)^2
\]

4. Prove that for all natural numbers, \( n \)
\[
1^3 + 2^3 + 3^3 + ... + n^3 = (1 + 2 + 3 + 4 + ... + n)^2
\][Hint: Use the result of Question 3].

5. Prove that for all natural numbers, \( n \)
\[
(1 \times 2) + (2 \times 3) + (3 \times 4) + ... + n(n + 1) = \frac{1}{3} n(n + 1)(n + 2)
\]

6. Prove that for all natural numbers, \( n \)
\[
(1 \times 2 \times 3) + (2 \times 3 \times 4) + (3 \times 4 \times 5) + ... + n(n + 1)(n + 2) = \frac{1}{4} n(n + 1)(n + 2)(n + 3)
\]

7. Show that for every natural number, \( n \)
\[
1 + r + r^2 + ... + r^n = \frac{1 - r^{n+1}}{1 - r} \quad (r \neq 1)
\][This is the geometric progression with the first term =1]

8. Show that for all natural numbers, \( n \)
\[
1 + 2 + 2^2 + ... + 2^n = 2^{n+1} - 1
\]

9. Prove that for all natural numbers, \( n \)
\[
1^3 + 3^3 + 5^3 + ... + (2n - 1)^3 = n^2 (2n^2 - 1)
\]

10. Prove that for all natural numbers, \( n \)
\[
1^4 + 2^4 + 3^4 + ... + n^4 = \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30}
\]
11. Prove that for all natural numbers, $n$

$$1^5 + 2^5 + 3^5 + \ldots + n^5 = \frac{n^2(n + 1)^2 \left(2n^2 + 2n - 1\right)}{12}$$