Exercise **1(g)**

1. Show that for all natural numbers, *n*:
$$2+4+6+\cdots+2n=n(n+1)$$

2. Prove that for all natural numbers, *n*:
$$(1\times 2)+(2\times 3)+(3\times 4)+\cdots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$$

3. Prove that for all natural numbers, *n*:
$$(1\times 2\times 3)+(2\times 3\times 4)+(3\times 4\times 5)+\cdots+n(n+1)(n+2)=\frac{1}{4}n(n+1)(n+2)(n+3)$$

4. Show that for all natural numbers, *n*:
$$1+2+2^2+\cdots+2^n=2^{n+1}-1$$

5. Show that for all natural numbers, *n*:
$$2+2^2+\cdots+2^n=2^{n+1}-2$$

6. Prove that for all natural numbers, *n*:
$$2+5+8+\cdots+(3n-1)=\frac{1}{2}n(3n+1)$$

7. Prove that for all natural numbers, *n*:
$$1^3+2^3+3^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$$

8. Prove that for all natural numbers, *n*:
$$1^3+2^3+3^3+\cdots+n^3=(1+2+3+4+\cdots+n)^2$$
[Hint: Use the result of Question 3].

9. Show that for every natural number, *n*:
$$1+r+r^2+\cdots+r^n=\frac{1-r^{n+1}}{1-r} \qquad (r\neq 1)$$
[This is the geometric progression with the first term equal to 1.]

10. Consider the following proposition $P(n)$ for the natural number *n*:
$$n+(n+1)=2n$$
Find the first error in the following proof.
“Proof”
Clearly the result is true for $n=1$.

Assume that $P(k)$ is true, that is $k+(k+1)=2k$

Then $P(k+1)$ is true because

$$(k+1)+(k+1)=2(k+1)$$

11. Let $P(n)$ be the false proposition

$$P(n):\quad n^2-n+41 \text{ is prime for all natural numbers } n$$

Determine the first error in the following derivation:

$P(1)$ is true. Assume the proposition is true for $n=k$, that is

$$p=k^2-k+41 \text{ is prime} \qquad (*)$$

Examining $P(k+1)$ we have

$$\begin{aligned}(k+1)^2-(k+1)+41 &= k^2+2k+1-k-1+41\\ &= k^2+k+41\\ &= \underbrace{k^2-k+41}_{p \text{ by } (*)}+2k\\ &= p+2k = \text{ prime}\end{aligned}$$

12. * Prove that for all natural numbers, n

$$1^3+3^3+5^3+\cdots+(2n-1)^3=n^2\left(2n^2-1\right)$$

13. *Prove that for all natural numbers, n

$$1^4+2^4+3^4+\cdots+n^4=\frac{n(n+1)(2n+1)\left(3n^2+3n-1\right)}{30}$$

14. *Prove that for all natural numbers, n

$$1^5+2^5+3^5+\cdots+n^5=\frac{n^2(n+1)^2\left(2n^2+2n-1\right)}{12}$$