Exercise **1(d)**

Throughout this exercise lower case letters such as $a, \quad b, \quad c, \ldots, n, \ldots$ *represent integers.*

1. Prove the following propositions:
(a) If n and m are even then their sum $n+m$ is even.
(b) If n and m are even then their subtraction $n-m$ is even.
(c) If n and m are odd then their subtraction $n-m$ is even.
(d) If n is an odd number then n^2 is also odd.
(e) If n is even and m is odd then their sum $m+n$ is odd.
(f) If n is odd and m is odd then their product nm is odd.
(g) If n is any integer and m is even then their product nm is even.

2. Prove the following propositions:
(i) n is odd $\Rightarrow$ $n+1$ is even
(ii) For any integer n we have $n(n+1)$ is even
[Hint: Use proposition (1.4)].

3. Prove that if n is odd then n^3-1 is even.
[Hint: Use the propositions proved in Question 1].

4. Prove the following propositions:
(a) $a \mid 0$ (b) $a \mid a$ (c) $1 \mid a$ (d) $a \mid a^2$
(e) $a \mid a^n$ where $n \geq 1$ is an integer
(f) $a \mid b$ and $a \mid c \Rightarrow a \mid (b+c)$
(g) $a \mid b$ and $a \mid c \Rightarrow a^2 \mid bc$
(h) $ac \mid bc \Rightarrow a \mid b$ where $c \neq 0$
(i) $a \mid b$ and $\mathrm{c} \mid d \Rightarrow ac \mid bd$

The remaining three questions are more difficult.

5. Prove the following proposition:
If n is odd then
(a) $8 \mid (n^2-1)$
(b) $32 \mid (n^2+3)(n^2+7)$

6. Show that if the last digit of an integer n is even then n is even.
[Hint: Write n as

$$n = a_m a_{m-1} a_{m-2} \ldots a_2 a_1 a_0 \quad (m > 1)$$

where $a_m, \quad a_{m-1}, \quad a_{m-2}, \ldots, \quad a_2, \quad a_1$ and a_0 are the digits of n. Note that a_0 is the last digit. Hence n is written as

$$n = (a_m \times 10^m) + (a_{m-1} \times 10^{m-1}) + (a_{m-2} \times 10^{m-2}) + \ldots + (a_2 \times 10^2) + (a_1 \times 10^1) + a_0 \text{]}$$

7. Show that if the last digit of an integer n is odd then n is odd.