Exercise **1(f)**

1. By constructing a truth table show that
$$\left[\text{not } (P \Rightarrow Q)\right] \equiv \left[P \wedge (\text{not } Q)\right] \qquad [\text{Equivalent}]$$

In each case prove the following statements by applying contradiction. In some cases it may be easier to do a direct proof but this is an exercise in proof by contradiction.

2. Prove the following proposition:
For every real number, x, there is a unique y such that
$$x + y = 0$$
[y is called the **additive inverse** of x].

3. Let x and y be real numbers. Prove that
$$xy = 0 \quad \Rightarrow \quad x = 0 \quad \text{or} \quad y = 0$$
In the remaining questions lower case letters represents an integer.

4. Prove that n^2 is odd $\Rightarrow$ n is odd.
[We have proved this result by contrapositive in Example 35. This time prove the result by contradiction and compare the two proofs].

5. Prove that n^3 is odd $\Rightarrow$ n is odd.

6. Prove that n^3 is even $\Rightarrow$ n is even.

7. Prove that ab is odd $\Rightarrow$ both a is odd and b is odd.

8. Prove that ab is even $\Rightarrow$ a is even or b is even.

9. Prove that $\sqrt{6}$ is irrational.

10. Prove that $\sqrt[3]{2}$ is irrational.

11. Prove that $\sqrt{17}$ is irrational. [Hint: Let p be prime and n be an integer greater than 1. Then $p \mid n^2 \quad \Rightarrow \quad p \mid n$].

12. Prove that there are **no** positive integer solutions such that
$$a^2 - b^2 = 1$$

13. Prove that the sum of a rational and irrational number is irrational.

14. Consider the triangle shown in Fig 5. Show that if $\angle B = \angle C$ then $AB = AC$

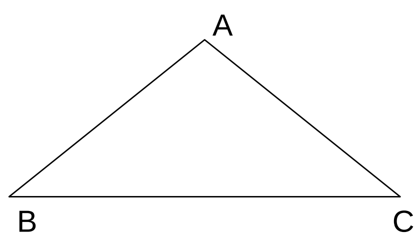

Fig 5