Exercise 1(f)

1. By constructing a truth table show that
\[ \text{not} \ (P \Rightarrow Q) = [P \land \text{not} \ Q] \]  
[Equivalent]

In each case prove the following statements by applying contradiction. In some cases it may be easier to do a direct proof but this is an exercise in proof by contradiction.

2. Prove the following proposition:
   For every real number, \( x \), there is a unique \( y \) such that \( x + y = 0 \)
   \( y \) is called the **additive inverse** of \( x \).

3. Let \( x \) and \( y \) be real numbers. Prove that \( xy = 0 \) \( \Rightarrow \) \( x = 0 \) or \( y = 0 \)
   *In the remaining questions lower case letters represents an integer.*

4. Prove that \( n^2 \) is odd \( \Rightarrow \) \( n \) is odd.
   \([\text{We have proved this result by contrapositive in Example 35. This time prove the result by contradiction and compare the two proofs}].\)

5. Prove that \( n^3 \) is odd \( \Rightarrow \) \( n \) is odd.

6. Prove that \( n^3 \) is even \( \Rightarrow \) \( n \) is even.

7. Prove that \( ab \) is odd \( \Rightarrow \) both \( a \) is odd and \( b \) is odd.

8. Prove that \( ab \) is even \( \Rightarrow \) \( a \) is even or \( b \) is even.

9. Prove that \( \sqrt{6} \) is irrational.

10. Prove that \( \sqrt{2} \) is irrational.

11. Prove that \( \sqrt{17} \) is irrational. \([\text{Hint: Let} \ p \text{ be prime and} \ n \text{ be an integer greater than 1. Then} \ p \mid n^2 \Rightarrow p \mid n].\)

12. Prove that there are no positive integer solutions such that \( a^2 - b^2 = 1 \)

13. Prove that the sum of a rational and irrational number is irrational.

14. Consider the triangle shown in Fig 5. Show that if \( \angle B = \angle C \) then \( AB = AC \)

![Fig 5](image-url)