Exercise 1(a)

1. Which of the following are propositions?
The ones that are propositions state whether they are true or false.
(a) $2 + 2 = 4$
(b) $2 + 2 = 3$
(c) All Swedish subjects have blonde hair.
(d) She looks beautiful.
(e) $x^2 - 1 = 0$
(f) Only fools fall in love.
(g) The equation $x - 5 = 6$ has the solution $x = 11$.
(h) Pigs will fly.
(i) Seat car has four airbags.
(j) Universities fix admissions.
(k) In some parts of Britain the temperature reached over $100^\circ F$ on the 10th August 2003.
(l) Sydney is the capital of Australia.
(m) The moon is made of cheese.
(n) BMW are excellent cars.

2. Negate the following propositions:
(i) Man can be pregnant.
(ii) Grass is green.
(iii) Lecturers annual salary is over £45 000.
(iv) $2 + 2 = 4$
(v) $16 < 3$
(vi) I was not born.
(vii) Today is Christmas day.
(viii) The lamp is not on.
(ix) There are no more than nine planets in our solar system.
(x) You are not telling the truth.
(xi) You have not passed the exam.
(xii) You are not undressed.
(xiii) There are integers $a$ and $b$ such that $\frac{a}{b} = \pi$
(xiv) There are integers $a$ and $b$ such that $\frac{a}{b} = e$

3. Let
$P$: You are dressed
$Q$: You are going to university

Write the following in symbolic form.
(a) You’re undressed and not going to university.
(b) If you are dressed then you are going to university.
(c) You are going to university but you are undressed.
(d) If you go to university then you are dressed.
(e) If you are not going to university then you are undressed.
(f) You are undressed or dressed and not going to university.
4. Write the following in words:

\[ x^2 - 9 = 0 \Rightarrow x^2 = 9 \]
\[ \Rightarrow x = \sqrt{9} \]
\[ \Rightarrow x = \pm 3 \]

5. Let \( P: x < 3, \) \( Q: x^2 < 9. \) Write a sentence for

(i) \( P \Rightarrow Q \) \hspace{1cm} (ii) \( Q \Rightarrow P \)

Do you think either of them, (i) and (ii), is true?

6. Let \( P: \) ABC is an equilateral triangle

\( Q: \) All the angles inside the triangle ABC are equal

Write a sentence for

(i) \( P \Rightarrow Q \) \hspace{1cm} (ii) \( Q \Rightarrow P \)

Are both these, (i) and (ii), true?

7. Let \( P: n \) is prime

\( Q: 2^n - 1 \) is prime

Write the following in words:

(i) \( P \Rightarrow Q \) \hspace{1cm} (ii) \( Q \Rightarrow P \)

Do you think either of them, (i) and (ii), is true?