

Complete solutions to Exercise 1(b)

1. The truth table is given by

q	p	$q \vee p$
T	T	T
T	F	T
F	T	T
F	F	F

By comparing with the truth table for $p \vee q$ we have

$$p \vee q \equiv q \vee p \quad [\text{Equivalent}]$$

2. Truth table is

q	p	$q \wedge p$
T	T	T
T	F	F
F	T	F
F	F	F

By comparing with the truth table for $p \wedge q$ we have

$$p \wedge q \equiv q \wedge p \quad [\text{Equivalent}]$$

3. (a)

p	$\neg p$	$\neg p \wedge p$
T	F	F
F	T	F

Clearly (not p) and p is going to give you false. That is

$$\neg p \wedge p \equiv F$$

(b)

p	$\neg p$	$\neg p \vee p$
T	F	T
F	T	T

Clearly (not p) or p is going to give you true (T). That is

$$\neg p \vee p \equiv T$$

(c)

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

Clearly $\neg\neg p \equiv p$

4. (a) $\neg(\neg(\neg p)) \equiv \neg p$

(b) $p \wedge p \equiv p$

(c) $p \vee p \equiv p$

(d) $\neg p \wedge (\neg p) \equiv \neg p$ [From 4(b)]

5. The truth table is

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Since the shaded columns are the same we can say $\neg(p \wedge q) \equiv \neg p \vee \neg q$

6. Truth table is:

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
F	T	T	T	F	F	F	F
T	F	F	F	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Shaded columns agree therefore

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

7. The rule is

$$\neg \neg \dots \neg p \equiv \begin{cases} p & \text{if the number of } \neg \text{'s is even} \\ \neg p & \text{if the number of } \neg \text{'s is odd} \end{cases}$$

Since we have 4 \neg 's in $\neg \neg \neg \neg p$ therefore using this rule we have

$$\neg \neg \neg \neg p \equiv p$$

which means the cup is full.