

Supplementary Exercises on Integration

1. Determine $\int_0^{\frac{\pi}{2}} x \sin^2(x) \, dx$.
2. Find $\int_1^2 (\ln(x))^2 \, dx$.
3. Evaluate $\int_0^{\pi} x^3 \sin(x) \, dx$.
4. Compute $\int_0^{\frac{1}{\sqrt{2}}} \frac{x^3}{(1-x^2)^{\frac{3}{2}}} \, dx$.
5. Compute $\int_0^3 \frac{x}{1+x^4} \, dx$.
6. Find the definite integral $\int_1^e x^{-\frac{1}{2}} \ln(x) \, dx$.
7. Determine $\int_1^e \frac{1}{x\sqrt{1-[\ln(x)]^2}} \, dx$.
8. Evaluate $\int_1^a \frac{e^{1/x}}{x^2} \, dx$ assuming $1 < a$.
9. Compute $\int_{-2}^2 \frac{1}{16-x^2} \, dx$.
10. Compute $\int_1^3 \frac{1}{\sqrt{4x-x^2}} \, dx$.
11. *Find the definite integral $\int_a^b \{(x-a)(b-x)\}^{-\frac{1}{2}} \, dx$ where $a < b$.
12. *Compute $\int_{-\pi}^{\pi} |\cos(x) + \sin(x)| \, dx$.

13. Investigate the existence of the following improper integrals, and evaluate those which exist:

(a) $\int_0^1 (1-x)\ln(x) dx$

(b) $\int_0^{+\infty} x^3 e^{-x^2} dx$

(c) $\int_0^{+\infty} \frac{\tan^{-1}(x)}{x^2} dx$

14. Determine the shaded area shown:

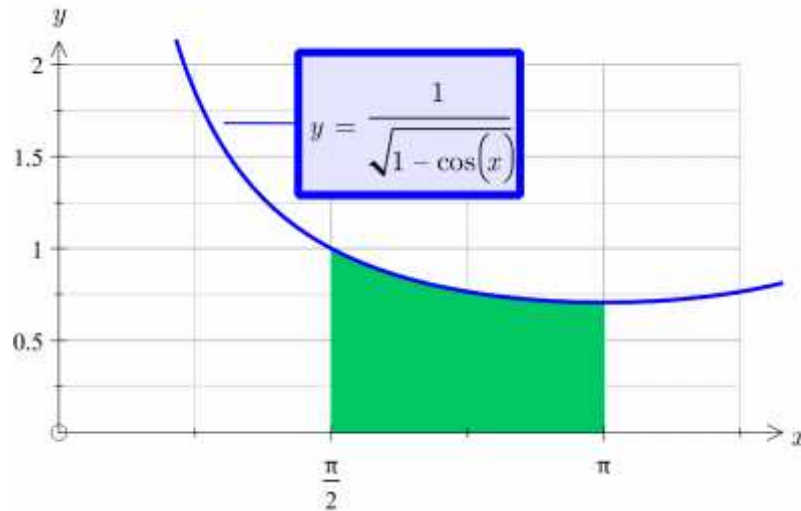


Figure 1

15. Evaluate $\int_0^a \frac{x^3}{(x^2+1)^3} dx$ where $a > 0$ by using the following substitutions:

(a) $u = x^2 + 1$

(b) $u = \tan^{-1}(x)$

Verify the equality of the results.

16. Let $I_n = \int_0^{\frac{\pi}{4}} \tan^n(x) dx$ where $n = 2, 3, 4, \dots$. Verify that

$$I_n + I_{n-2} = \frac{1}{n-1}$$

17. Show that

$$\int_0^{+\infty} x^n e^{-x^2} dx = \frac{1}{2}(n-1) \int_0^{+\infty} x^{n-2} e^{-x^2} dx \text{ where } n = 2, 3, 4, \dots$$

And hence evaluate $\int_0^{+\infty} x^5 e^{-x^2} dx$.

18. For every integer $n \geq 1$ define $I_n = \int_0^{\frac{\pi}{2}} \frac{\cos(nx) - 1}{\sin(x)} dx$. By considering

$$I_n - I_{n-2}$$

Or otherwise, evaluate I_3 and I_4 .

Brief Solutions

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|---|---|--------------------|-----------------------------|
| 1. $\frac{1}{4} \left(\frac{\pi^2}{4} + 1 \right)$ | 2. $2 \left[(\ln(2)) - 1 \right]^2$ | 3. $\pi^3 - 6\pi$ | 4. $\frac{3}{\sqrt{2}} - 2$ |
| 5. $\frac{1}{2} \tan^{-1}(9)$ | 6. $4 - 2e^{\frac{1}{2}}$ | 7. $\frac{\pi}{2}$ | 8. $e - e^{\frac{1}{a}}$ |
| 9. $\frac{1}{4} \ln(3)$ | 10. $\frac{\pi}{3}$ | 11. π | 12. $4\sqrt{2}$ |
| 13. (a) $-\frac{3}{4}$ | (b) $\frac{1}{2}$ | (c) diverges | |
| 14. $-\sqrt{2} \left[\ln \left(\tan \left(\frac{\pi}{8} \right) \right) \right]$ | 15. (a) and (b) $\frac{a^4}{\left[2(a^2 + 1) \right]^2}$ | 17. 1 | |
| 18. $I_3 = -2 - \ln(2)$, $I_4 = -\frac{8}{3}$ | | | |