Exercise 17(f)

1. Obtain the Fourier series of the function $h(t)$ of Example 13.

2. The function $f(t)$ is such that

$$f(t) = 2 - t \quad \text{for} \quad 0 \le t < 2$$

The graph of this is

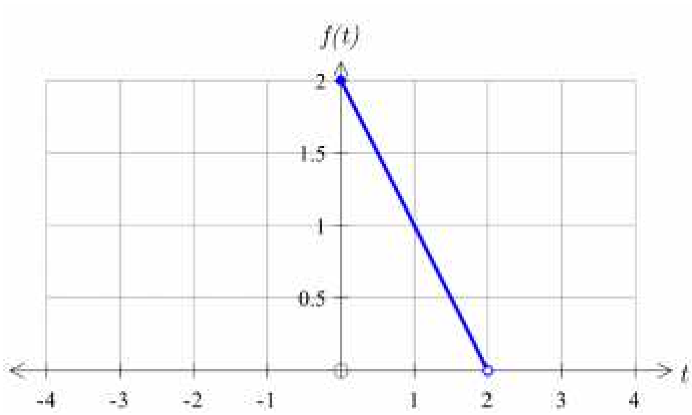

Figure 53

(i) Sketch the graph of $f(t)$ and find its Fourier series in the case $f(t)$ is even.

(ii) From your Fourier series deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

3. Consider the following waveform which has period 2π:

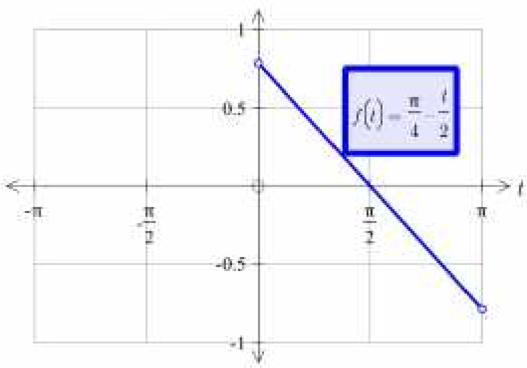

Figure 54

(i) Sketch the graph of $f(t)$ and determine its Fourier series in the case $f(t)$ is odd.

(ii) From this series deduce that

$$\frac{\pi}{8} = \frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + \cdots$$

4. The half range function $f(t)$ with period 2π is given by

$$f(t) = t \qquad \text{if } 0 \le t < \pi$$

(a) Sketch the graph of $f(t)$ and find its Fourier series in the case it is an *odd* function.

(b) Sketch the graph of $f(t)$ and find its Fourier series in the case it is an even function.

(c) Sketch the graph of $f(t)$ and find its Fourier series in the case it contains both sine and cosine terms.

5. Find the half-range Fourier cosine series of the function defined in Example 14 and sketch the graph.

6. *Let $f(t) = \sin(t)$, $0 \le t \le \pi$ be the half range function. The graph below in Fig. 55 is of the extended function $f(t)$ so that it becomes an even function: (This is called the full rectified sine wave.)

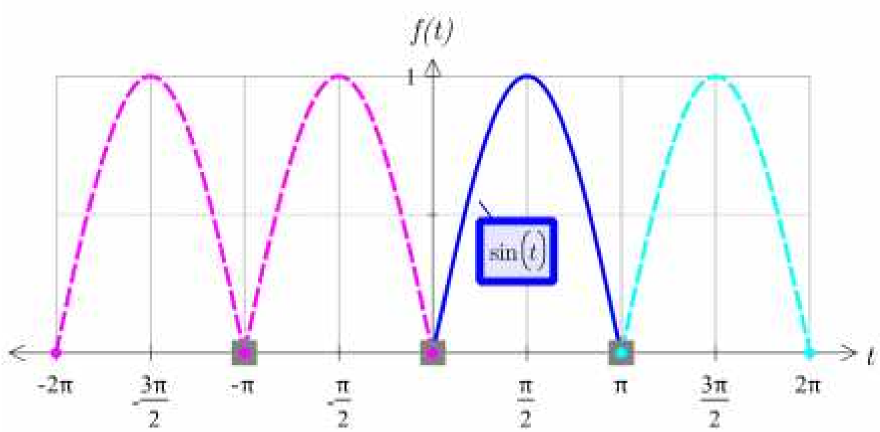

Figure 55

(i) Show that the Fourier series of $f(t)$ in a Fourier cosine series is given by:

$$f(t) = \frac{2}{\pi} - \frac{4}{\pi}\left[\frac{\cos(2t)}{2^2 - 1} + \frac{\cos(4t)}{4^2 - 1} + \frac{\cos(6t)}{6^2 - 1} + \cdots\right]$$

(ii) By using an appropriate value for t deduce that

$$\frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \frac{1}{6^2 - 1} + \cdots = \frac{1}{2}$$

(iii) Find the Fourier sine series of $f(t)$.

7. Let $f(t)=\sin(t)$, $0 \le t \le \pi$. Extend the definition of this function so that it has a period 2π and the resulting Fourier series has both sine and cosine terms.

(i) Sketch the graph of the extended $f(t)$.

(ii) Find the Fourier series of this extended function.

You may find the following result helpful:

$$\int_0^\pi \sin(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{if } m \ne n \\ \pi/2 & \text{if } m = n \end{cases}$$

8. The following is called the half wave rectified sine waveform:

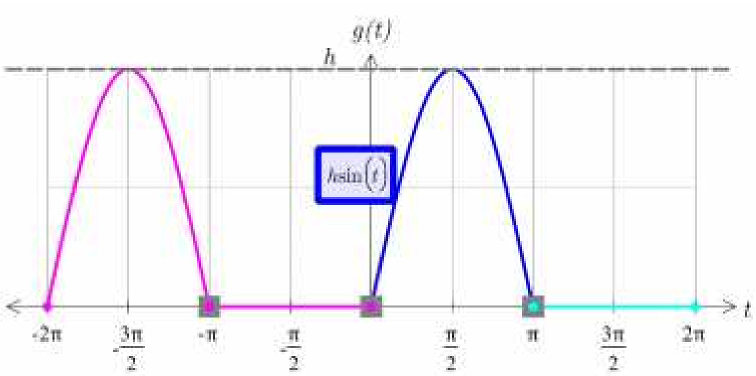

Figure 56

Show that the Fourier series of this waveform is given by

$$g(t)=\frac{h}{\pi}-\frac{2h}{\pi}\left[\frac{\cos(2t)}{2^2-1}+\frac{\cos(4t)}{4^2-1}+\frac{\cos(6t)}{6^2-1}+\cdots\right]+\frac{h}{2}\sin(t)$$

Brief Solutions

1. $h(t)=1$

2. (i) $1+\frac{8}{\pi^2}\left[\cos\left(\frac{\pi t}{2}\right)+\frac{\cos(3\pi t/2)}{3^2}+\frac{\cos(5\pi t/2)}{5^2}+\cdots\right]$

3. (i) $\frac{1}{2}\sin(2t)+\frac{1}{4}\sin(4t)+\frac{1}{6}\sin(6t)+\frac{1}{8}\sin(8t)+\cdots$

4. (a) $2\left[\sin(t)-\frac{\sin(2t)}{2}+\frac{\sin(3t)}{3}-\frac{\sin(4t)}{4}+\frac{\sin(5t)}{5}-\cdots\right]$

(b) $\frac{\pi}{2}-\frac{4}{\pi}\left[\cos(t)+\frac{\cos(3t)}{3^2}+\frac{\cos(5t)}{5^2}+\frac{\cos(7t)}{7^2}+\cdots\right]$

(c) If $f(t)=\begin{cases} t & \text{if } 0 \le t < \pi \\ 0 & \text{if } -\pi < t \le 0 \end{cases}$ then

$$f(t) = \frac{\pi}{4} - \frac{2}{\pi}\left[\cos(t) + \frac{\cos(3t)}{3^2} + \frac{\cos(5t)}{5^2} + \cdots\right] + \left[\sin(t) - \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} - \cdots\right]$$

5. $\pi - \dfrac{8}{\pi}\left[\cos(t) + \dfrac{\cos(3t)}{3^2} + \dfrac{\cos(5t)}{5^2} + \dfrac{\cos(7t)}{7^2} + \cdots\right]$

6. (iii) $f(t) = \sin(t), \quad 0 \le t \le 2\pi$

7. $\dfrac{1}{\pi} - \dfrac{2}{\pi}\left[\dfrac{\cos(2t)}{2^2 - 1} + \dfrac{\cos(4t)}{4^2 - 1} + \dfrac{\cos(6t)}{6^2 - 1} + \cdots\right] + \dfrac{1}{2}\sin(t)$