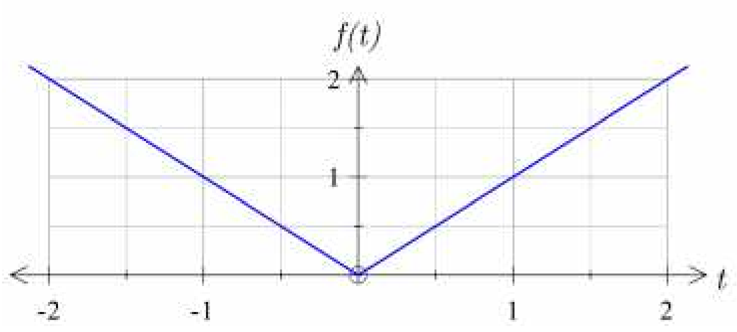
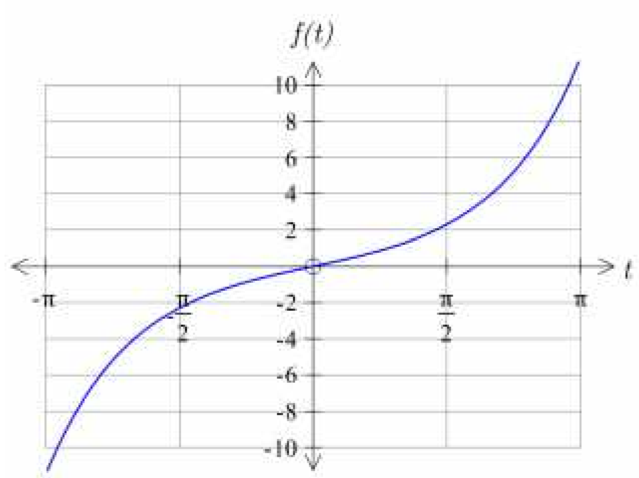
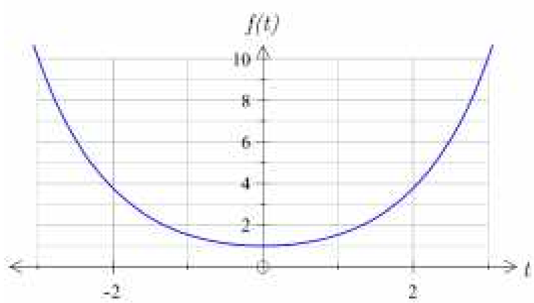
Exercise 17(d)

1. Determine whether the following waveforms $f(t)$ are odd, even or neither:

(a)


(b)


(c)


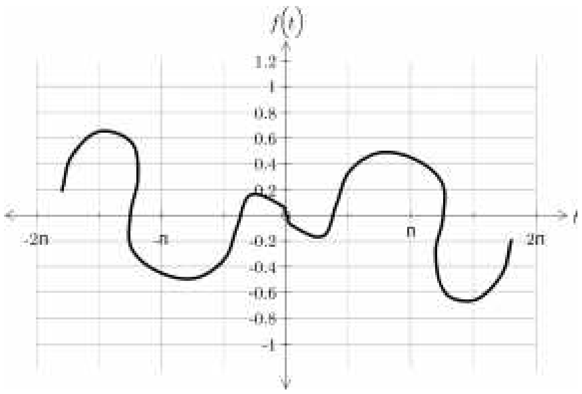
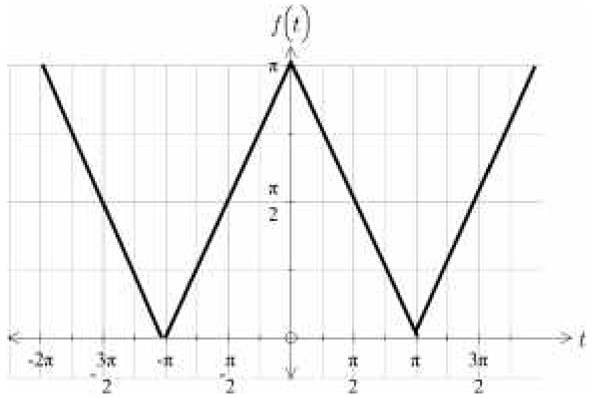
(d)


Figure 32

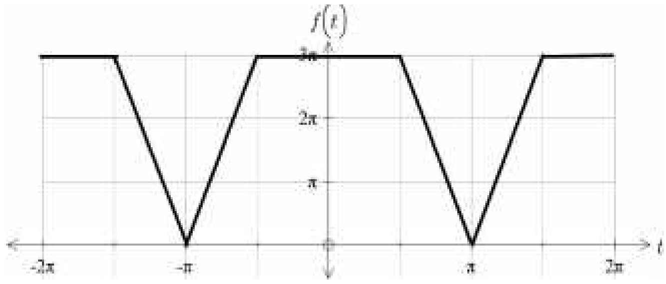
2. Determine whether the following functions are odd, even or neither:

   (a) e^t (b) $\tan(t)$ (c) $\dfrac{1}{1+t^2}$ (d) $e^{-t}\sin(t)$

3. Determine the constant term A_0 of the Fourier series for the following waveforms which have a period of 2π:



(a)



(b)

Figure 33

4. (i) Determine the Fourier series of the waveform shown in Fig. 33(a).
   (ii) Deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$$

5. Let k be any integer. Show the following results:
   (a) $\cos(kt)$ is even function.
   (b) $\sin(kt)$ is an odd function.

6. **(a) For an *odd* function, $f(t)$, with period 2π show that

$$A_0 = A_k = 0 \text{ and } B_k = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(kt)\,\mathrm{d}t$$

   **(b) For an *even* function, $f(t)$, with period 2π show that

$$A_k = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(kt)\,\mathrm{d}t \text{ and } B_k = 0$$

[Hint: Consider change of variable $t = -x$ and use the result

$$\int_a^b f(y)\,\mathrm{d}y = -\int_b^a f(y)\,\mathrm{d}y\,]$$

7. Consider the following waveform of period 2π:

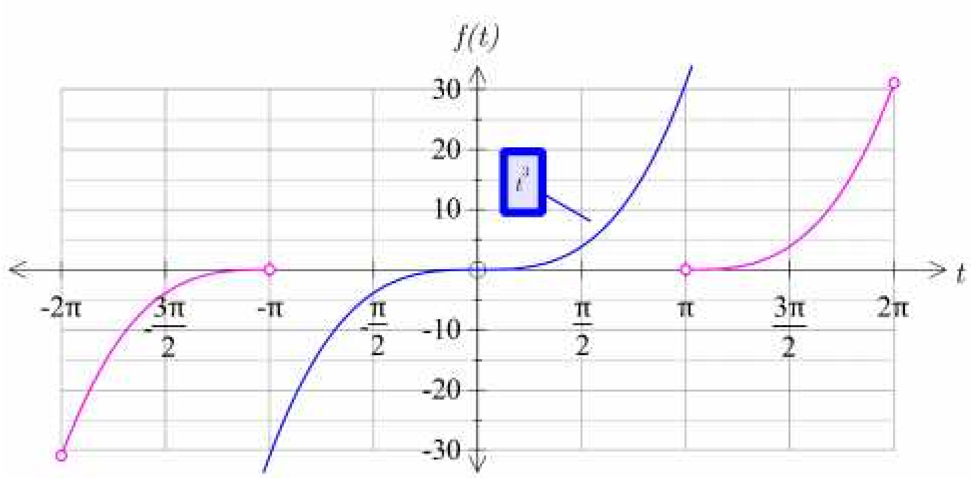


Figure 34

Show that the Fourier series of this waveform is given by

$$f(t) = -2\left[(6-\pi^2)\sin(t) - (6-4\pi^2)\frac{\sin(2t)}{2^3} + (6-9\pi^2)\frac{\sin(3t)}{3^3} - (6-16\pi^2)\frac{\sin(4t)}{4^3} + (6-25\pi^2)\frac{\sin(5t)}{5^3} - \ldots\right]$$

You may find the following result helpful:

$$\int_0^{\pi} x^3 \sin(nx)\,\mathrm{d}x = \frac{(6\pi - n^2\pi^3)}{n^3}\cos(n\pi)$$

8. Show that the Fourier series of the following square wave of height h and period 2π:

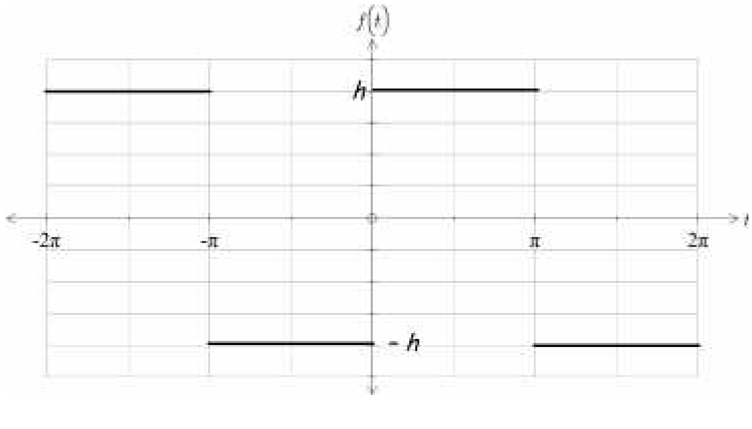


Figure 35

Is given by $f(t)=\dfrac{4h}{\pi}\left[\sin(t)+\dfrac{\sin(3t)}{3}+\dfrac{\sin(5t)}{5}+\dfrac{\sin(7t)}{7}+\cdots\right].$

9. Show that the Fourier series of the following square wave of amplitude h and period 2π:

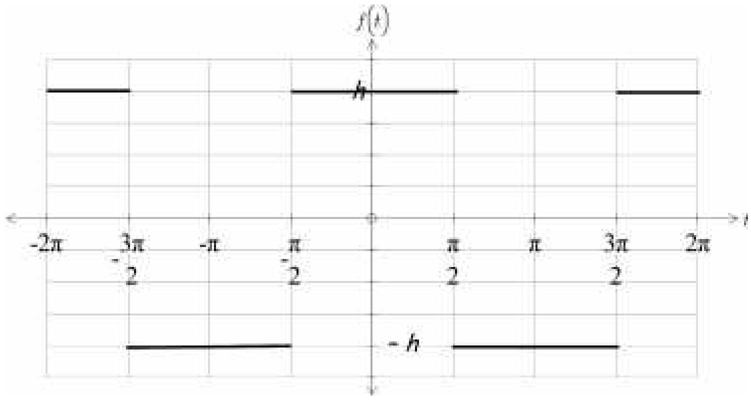


Figure 36

Is given by $f(t)=\dfrac{4h}{\pi}\left[\cos(t)-\dfrac{\cos(3t)}{3}+\dfrac{\cos(5t)}{5}-\dfrac{\cos(7t)}{7}+\cdots\right].$

By using $h=\pi$ deduce that

$$\frac{\pi^2}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\cdots$$

10. * Consider the following waveform which has a period of 2π:

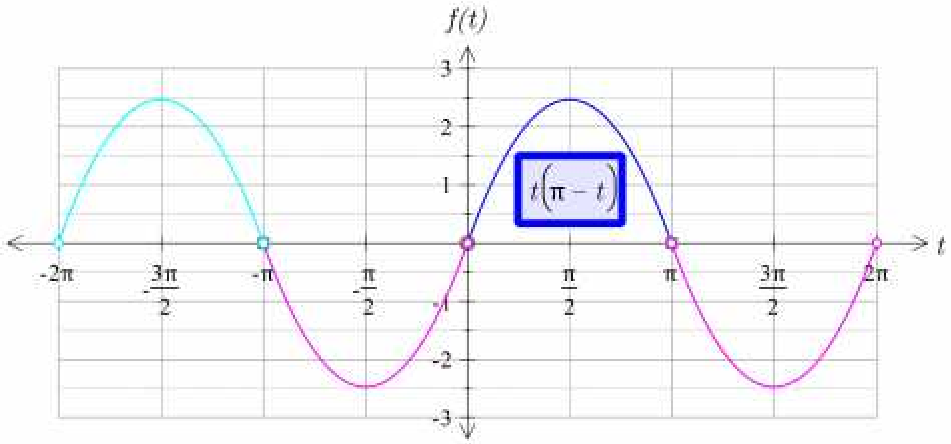


Figure 37

(i) Show that the Fourier series of $f(t)$ is given by

$$f(t)=\frac{8}{\pi}\left[\sin(t)+\frac{\sin(3t)}{3^3}+\frac{\sin(5t)}{5^3}+\frac{\sin(7t)}{7^3}+\cdots\right]$$

(ii) Deduce that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

Brief Solutions

1. (a) even (b) odd (c) even (d) odd
2. (a) neither (b) odd (c) even (d) neither
3. (a) $\frac{\pi}{2}$ (b) $\frac{9\pi}{4}$
4. $f(t) = \frac{\pi}{2} + \frac{4}{\pi}\left[\cos(t) + \frac{\cos(3t)}{9} + \frac{\cos(5t)}{25} + \frac{\cos(7t)}{49} + \cdots\right]$