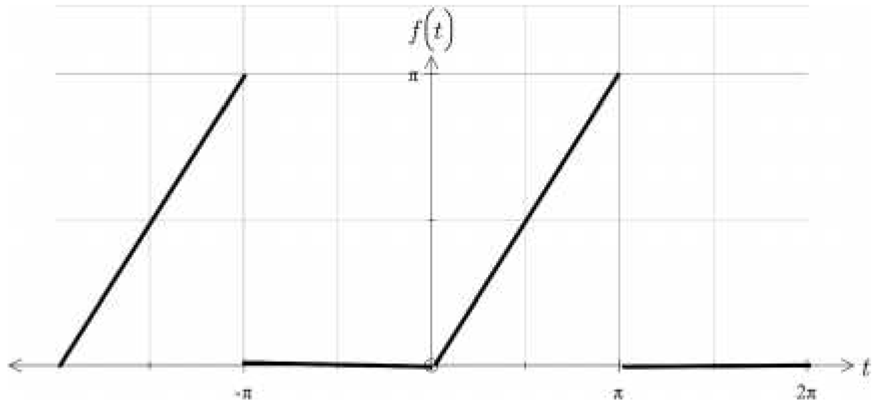
Exercise 17(c)

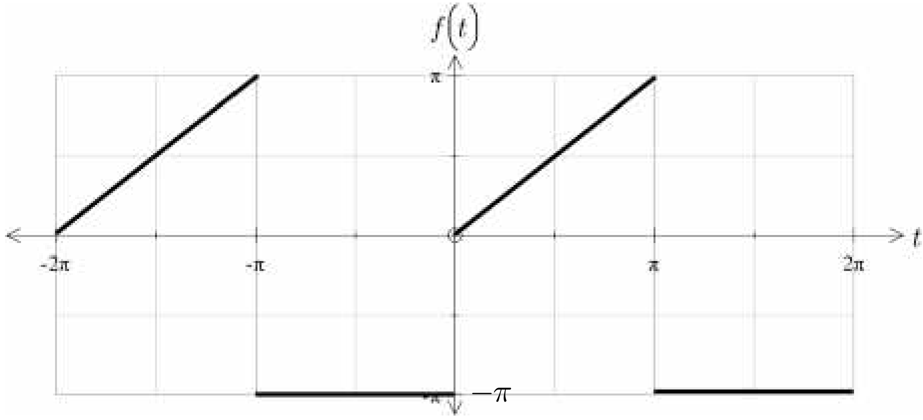
1. (a) (i) Find the Fourier series of the following waveform which has a period of 2π:



(ii) By substituting an appropriate value for t into your Fourier series deduce that

$$\frac{\pi^2}{8} = 1 + \frac{1}{9} + \frac{1}{25} + \frac{1}{49} + \frac{1}{81} + \cdots$$

(b) Find the Fourier series of the following waveform which has a period of 2π:



2. (i) Sketch the following function which has a period of 2π:

$$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ 3t & 0 < t < \pi \end{cases}$$

(ii) Determine the Fourier series of this function $f(t)$.

3. (i) Let $f(t)$ have a period of 2π and is such that

$$f(t) = \begin{cases} t^2 & \text{when } 0 < t < 2\pi \\ 2\pi^2 & \text{when } t = 0 \text{ and } t = 2\pi \end{cases}$$

Find its Fourier series.

(ii) By substituting an appropriate value for t into your Fourier series deduce that

$$\zeta(2) = \sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots = \frac{\pi^2}{6}$$

4. Determine the Fourier series of the following function with period 2π:

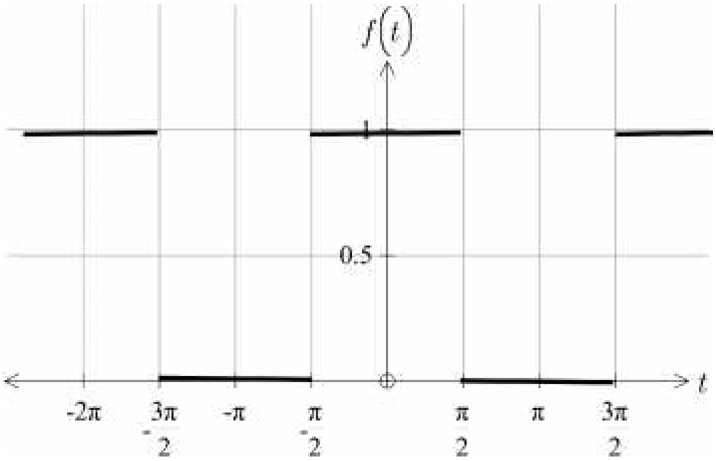
$$f(t) = t^2 + t, \quad 0 < t < 2\pi$$

[Hint: Use your solution to question 3 and the Fourier series obtained in Example 5.]

5. (i) Sketch the graph of $f(t) = t, \ -\pi < t < \pi$ which has a period of 2π.

(ii) Find the Fourier series of $f(t)$.

6. (i) Determine the Fourier series of the following waveform which has a period of 2π:



(ii) Deduce

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

7. (i) Let a pulse waveform $f(t)$ have period 2π and is given by

$$f(t) = \begin{cases} 0 & \text{when } -\pi < t < 0 \\ 1 & \text{when } \quad 0 < t < \pi/3 \\ 0 & \text{when } \pi/3 < t < \pi \end{cases}$$

Sketch of graph of $f(t)$ between -2π and 2π.

(ii) Determine the Fourier series of $f(t)$.

Brief Solutions

1. (a) $\frac{\pi}{4} - \frac{2}{\pi}\left(\cos(t) + \frac{\cos(3t)}{9} + \frac{\cos(5t)}{25} + \cdots\right) + \left(\sin(t) - \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} - \cdots\right)$

   (b) $$-\frac{\pi}{4} - \frac{2}{\pi}\left(\cos(t) + \frac{\cos(3t)}{9} + \frac{\cos(5t)}{25} - \cdots\right)$$
   $$+ 3\left(\sin(t) + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \cdots\right) - \left(\frac{\sin(2t)}{2} + \frac{\sin(4t)}{4} + \frac{\sin(6t)}{6} + \cdots\right)$$

2. (ii) $\frac{3\pi}{4} - \frac{6}{\pi}\left(\cos(t) + \frac{\cos(3t)}{9} + \frac{\cos(5t)}{25} + \cdots\right) + 3\left(\sin(t) - \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} - \frac{\sin(4t)}{4} + \cdots\right)$

3. $\frac{4\pi^2}{3} + 4\left[\cos(t) + \frac{\cos(2t)}{4} + \frac{\cos(3t)}{9} + \ldots\right] - 4\pi\left[\sin(t) + \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} + \ldots\right]$

4. $$\left(\frac{4\pi^2}{3} + \pi\right) + 4\left[\cos(t) + \frac{\cos(2t)}{4} + \frac{\cos(3t)}{9} + \ldots\right]$$
   $$-(4\pi + 2)\left[\sin(t) + \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} + \ldots\right]$$

5. (ii) $2\left[\sin(t) - \frac{\sin(2t)}{2} + \frac{\sin(3t)}{3} - \frac{\sin(4t)}{4} + \ldots\right]$

6. $\frac{1}{2} + \frac{2}{\pi}\left[\cos(t) - \frac{\cos(3t)}{3} + \frac{\cos(5t)}{5} - \right]$

7. (ii) $$\frac{1}{6} + \frac{\sqrt{3}}{2\pi}\left[\cos(t) + \frac{\cos(2t)}{2} - \frac{\cos(4t)}{4} - \frac{\cos(5t)}{5} + \cdots\right]$$
   $$+ \frac{1}{\pi}\left[\frac{\sin(t)}{2} + \frac{3\sin(2t)}{4} + \frac{2\sin(3t)}{3} + \frac{3\sin(4t)}{8} + \frac{\sin(5t)}{10} + \cdots\right]$$